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Topology synthesis of multi-material compliant mechanisms with a Sequential Element Rejection and Admission method

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ABSTRACT

The design of multi-material compliant mechanisms by means of a multi Sequential Element Rejection and Admission (SERA) method is presented in this work. The SERA procedure was successfully applied to the design of single-material compliant mechanisms. The main feature is that the method allows material to flow between different material models. Separate criteria for the rejection and admission of elements allow material to redistribute between the predefined material models and efficiently achieve the optimum design. These features differentiate it to other bi-directional discrete methods, making the SERA method very suitable for the design of multi-material compliant mechanisms. Numerous examples are presented to show the validity of the multi SERA procedure to design multi-material compliant mechanisms.

1. Introduction

Compliant mechanisms can be defined as monolitic structures that rely on its own elastic deformation to achieve force and motion transmission [1]. They have undergone considerable development since the introduction of both advanced materials and the field of MicroElectroMechanical Systems (MEMS). These submillimeter mechanical systems are the most promising application area of compliant mechanisms. They are coupled with electronic circuits and manufactured using etching techniques and surface micromachining processes from the semiconductor industry [2]. The use of hinges, bearings and assembly processes are prohibitive due to their small size, and must be built and designed as compliant mechanisms etched out of a single piece.

The most widely studied compliant mechanisms are single-material devices. Originally accomplished by trial and error methods, researchers took an interest in the systematic design of this type of compliant mechanisms by means of topology optimization techniques [3–5]. The main advantage of these techniques was that the optimum design was automatically suggested for a target volume fraction for a prescribed design domain, boundary conditions and functional specifications. There was no need to predetermine the number of links or the location of the flexural joints in the device [6].

The optimization methods used for this purpose were diverse. Among others, the homogenization method [3,7], the SIMP method [5], the Genetic Algorithms [8], the Level Set methods [9] and, more recently, the SERA method [10].

During the last decade, the design of devices with multiple materials gained popularity with the recent development of manufacturing methods. It is the case of the coextrusion of plastics, the shape deposition manufacturing [11], or the layered manufacturing with embedded components [12].

As a result, some of the methods applied to single-material compliant mechanisms were also applied to the design of multi-material compliant mechanisms. Sigmund [13] performed topological synthesis of electrothermal actuators with nonlinear deformation and multiple materials and output ports. In this work, Sigmund studied the effect on the mechanisms performance of using two materials for thermal and electrothermal actuators. The conclusion was that the use of two materials was beneficial only in some cases and that those gains were, in many cases, insignificant. Yin and Ananthasuresh [14] proposed a peak function material interpolation scheme to incorporate multiple materials to the design of compliant mechanisms without increasing the number of design variables. In the two aforementioned works, the optimization methods used were gradient based with algorithms comprising the optimality criterion [14] or the method of moving asymtotes [13].

More recently developed methods were also applied to the design of multi-material compliant mechanisms. Wang et al. [15]

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extended the Level Set approach to the design of monolithic compliant mechanisms made of multiple materials. The mechanical advantage of the mechanisms was used as the objective function. Saxena [16] used Genetic Algorithms to compute the synthesis of compliant mechanisms with multiple materials and output displacements. Geometrically nonlinear analysis was used and the implementation was accomplished usign frame finite elements.

This research presented in this article focuses on the design process of multi-material compliant mechanisms which have already been used in several applications such as piezoelectric devices [24], bimorph actuators [13], and grippers and clamping devices [15,25]. For these applications, the need to use multi-materials for the design of the compliant mechanisms arose because: (a) one of the materials was more expensive; (b) there was a requirement for a stiffer internal mechanism structure with a flexible exterior shell made from a weaker material; (c) there was a requirement for the mechanism to be porous; (d) there was an electrical insulation requirements so regions of the mechanism had to have an electrically non-conductive phase; and (e) the esthetic requirements of the mechanism as specified by the designer, meant that more than one material had to be used.

The aim of this article is to present a generalized formulation for the design of multi-material compliant mechanisms with the use of a Sequential Element Rejection and Admission (SERA) method [17,18]. This method was succesfully applied to the design of single-material compliant mechanisms [10]. The procedure considers two separate criteria for the rejection and admission of elements and material was redistributed between two material models: “real” material and “virtual” material with negligible stiffness. This feature of the SERA method makes it ideally suited for the design of multi-material compliant mechanisms. The formulation presented here is an extension from the one used for single-material compliant mechanisms [10] where the objective was to maximize the Mutual Potential Energy of the mechanism under a constraint in the target volume fraction. Benchmark examples are used to demonstrate the validity of the proposed method to design multi-material compliant mechanisms.

2. Problem formulation of a multi-material compliant mechanism

A multi-material compliant mechanism is required to meet the flexibility and stiffness requirements in order to withstand the applied loads and produce the predefined displacement transmission. Fig. 1 shows such a multi-material compliant mechanism domain Ω. It is subjected to a forces F_{in} at the input port P_{in} and is supposed to produce an output displacements Δ_{out} at the output port P_{out}.

The goal of topology optimization for multi-material compliant mechanisms is to obtain the optimum design that converts the input work into an output displacement in a predefined direction. The mathematical formulation of this work is expressed as the maximization of the Mutual Potential Energy (MPE) (Eq. (1)) subjected to M constraints on the target volume fraction of the M materials, V_m^* (Eq. (2)). The summation of target volume fractions must be the unit (Eq. (3)) as each element can only be in one material model:

$$\max \text{MPE} \tag{1}$$

$$\text{subjected to : for } m=0,\dots,M$$

$$\sum_{e=1}^{N} \rho_m^e \cdot \frac{V_m^e}{V_{Tot}} \le V_m^*, \quad \rho_m^e = \{\rho_{\min}, 1\}, \; e=1,\dots,N \tag{2}$$

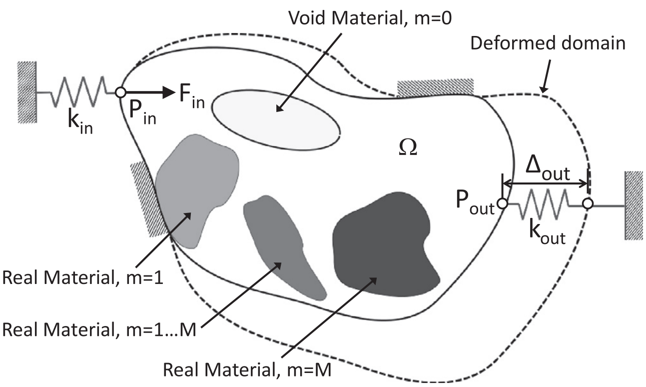

Fig. 1. Problem definition of a multi-material compliant mechanism.

$$\sum_{m=0}^{M} V_m^* = 1, \quad m=0,\dots,M \tag{3}$$

where ρ_m^e is the density of the eth finite element and material m, V_m^e is the volume of the eth element and material m, V_{Tot} is the total volume for the domain, M is the number of materials, N is the number of finite elements and ρ_{min} is the minimum density considered, a typical value of which is 10^{-4}. Void material is represented with $m=0$.

The MPE (Eq. (4)) [19] was defined as the deformation at a prescribed output port in a specified direction. To obtain the MPE, two load cases are calculated: (1) the input force case, where the input force F_{in} is applied to the input port P_{in}, named with the subscript 1 in (4) and (5) and Fig. 2a and (2) the pseudo-force case, where a unit force is applied at the output port P_{out} in the direction of the desired displacement, named with the subscript 2 in (4) and (6) and Fig. 2b:

$$\text{MPE} = \boldsymbol{U}_2^T \cdot \boldsymbol{K} \cdot \boldsymbol{U}_1 \tag{4}$$

$$\boldsymbol{K} \cdot \boldsymbol{U}_1 = \boldsymbol{F}_1 \tag{5}$$

$$\boldsymbol{K} \cdot \boldsymbol{U}_2 = \boldsymbol{F}_2 \tag{6}$$

where $\boldsymbol{K}$ is the global stiffness matrix of the structure; $\boldsymbol{F}_1$ is the nodal force vector which contains the input force F_{in}; $\boldsymbol{F}_2$ is the nodal force vector which contains the unit output force F_{out}; and $\boldsymbol{U}_1$ and $\boldsymbol{U}_2$ are the displacement fields due to each load case.

The global stiffness matrix $\boldsymbol{K}$ is expressed by the density of the eth finite element and the elemental stiffness matrix:

$$K = \sum_{e}^{N} \rho_m^e \cdot K_m^e(E_m, \upsilon_m), \quad m=0,\dots,M, \;\; e=1,\dots,N \tag{7}$$

where K_m^e is the elemental stiffness matrix of the eth element, which depends on the Young modulus E_m and Poisson ratio υ_m of the m isotropic material.

The definition of the stiffness at the input and output ports is done in this work with the use of the spring model of Fig. 1. The artificial input spring k_{in} together with an input force F_{in} simulates the input work of the actuator. The resistance to the output displacement is modeled with a spring of stiffness k_{out}. This allows the displacement amplification to be controlled by specifying different values of the input and output springs.

As part of the optimization process, a sensitivity analysis is carried out to provide information on how sensitive the objective function is to small changes in the design variables. The derivative of the MPE with respect to the element density is given as

$$\frac{\partial \text{MPE}}{\partial \rho_e} = \frac{\partial}{\partial \rho_e}(\boldsymbol{U}_2^T \cdot \boldsymbol{K} \cdot \boldsymbol{U}_1) = \left(\frac{\partial \boldsymbol{U}_2^T}{\partial \rho_e} \cdot \boldsymbol{K} \cdot \boldsymbol{U}_1 + \boldsymbol{U}_2^T \cdot \frac{\partial \boldsymbol{K}}{\partial \rho_e} \cdot \boldsymbol{U}_1 + \boldsymbol{U}_2^T \cdot \boldsymbol{K} \cdot \frac{\partial \boldsymbol{U}_1}{\partial \rho_e} \right) \tag{8}$$