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Modified Hermitian cubic spline wavelet on interval finite element for wave propagation and load identification

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Accuracy and efficiency are significant factors in wave propagation and load identification of mechanical structure. By introducing modified Hermitian cubic spline wavelets on interval (HCSWI), a multi-scale wavelet-based numerical method is proposed. The present method can avoid the boundary problem of the original Hermitian interpolation wavelet. A modified Hermitian interpolation wavelet base can get transformation matrix, so the modified Hermitian wavelet finite element is proposed in this paper. Positive question-wave propagation and inverse question-load identification is verified by this means. The modified Hermitian wavelet finite element involves wave propagation and load identification in rod and Timoshenko beam which are obtained and then compared with results calculated by traditional finite element method (TFEM) and B-spline wavelet on interval (BSWI) finite element. The results indicate that the present method for wave propagation and load identification has higher precision and costs less time on mechanical structure.

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1. Introduction

Wave propagation and load identification are the key subjects of intensive investigations in mechanical engineering over the years [1,2]. Wave propagation techniques providing an efficient and accurate procedure have been of great interest to many researchers in mechanical structures [3–5]. However, a so-called "short wave problem" for finite element based techniques becomes one of the biggest obstacles which must be overcome [6]. Zienkiewicz and Taylor noticed the "rule of thumb" that there should be at least 10 nodes per wave length [6]. Babuska et al. found an effect called "pollution error", which makes even more astronomical computing requirements than that of the "rule of thumb" [7,8]. Thus, accuracy and efficiency are importantly and greatly required for finite element techniques when they are used in high frequency wave propagation simulation [9]. Manktelow et al. pointed out that nonlinear dispersion through an integrated commercial software environment which enables exploration and optimization of geometrically-complex structures [10]. Pahlavan et al. presented a novel and generic formulation of the wavelet-based spectral finite element approach, which is applicable to elastic wave propagation problems [11]. Yang et al. analyzed elastic wave propagation in arches using a B-spline wavelet on interval finite element [12].

Accurate and reliable data on mechanical structure loads are highly necessary not only to design and development but also to strength and rigidity specifications for mechanical structures [13]. But due to the complexity of structure and loading conditions, a traditional way of obtaining the applied force from direct measurement may not always be possible because of difficulties in sensor placement or other practical problems [14]. Rowley described the solved loads from measured responses based on moving force identification algorithm [15]. Therefore, it is crucial to study proper algorithm with load identification. Maes et al. presented an analytical method to estimate the axial force in a beam member. The method accounts for bending stiffness effects and for the rotational inertia and shear deformation of the beam member [16]. Li et al. used system characteristics and responses to calculate loads, which are based on wavelet multi-resolution analysis [17]. Gupta presented a time domain technique for estimating dynamic loads acting on a structure from strain time response [18].

Finite element method (FEM) has been playing an important role in many engineering fields. Wave propagation and load identification techniques adapting to more complex structures became possible and available because of the use of Finite element method [19]. However, for many complicated problems, TFEM has some disadvantages, such as low efficiency, insufficient accuracy, slow convergence to correct solutions etc. Recently, wavelets have been applied to obtain representations of integral and differential operators in many physical problems [20,21]. And because wavelets have the properties of multi-resolution analysis, they provide

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a natural mechanism for decomposing the solution into a set of coefficients. Wavelet analysis numerical methods can be viewed as those interpolating functions, similar to those used in signal and image processing. Basu indicated that the finite difference and Ritz-type methods had been largely replaced by the FEM, the boundary element method, the meshless method, and in the near future it might be the turn for the wavelet-based numerical method [22]. Since B-spline wavelets have explicit expressions, high degree of accuracy and high efficiency, numerous researchers focus on the B-spline wavelet on interval (BSWI) finite element method [23,24]. But compared with BSWI, which is formed to recalculate the function to improve the precision of the new scales from the original scale function, HCSWI, however, has prominent advantages of improving the precision by adding the appropriate wavelet function. Based on modified HCSWI, this paper presents a multi-scale wavelet-based numerical method, a modified Hermitian interpolation wavelet base, avoiding the boundary problem of the original Hermitian interpolation wavelet [25], and proposing the modified Hermitian wavelet finite element.

In the present work, an effective wavelet numerical method is proposed based on wavelet bases of modified Hermitian cubic spline wavelets on interval [26] to analyze wave propagation and load identification of rod and beam. For the orthogonal characteristic of the wavelet bases with respect to the given inner product, the corresponding multi-scale solution equation will be decoupled across levels totally or partially and it suits for the nesting approximation. Some numerical examples indicate that the proposed method has better precision in analyzing mechanical structure [27].

2. Hermitian cubic splines on interval

Wang constructed cubic spline wavelet bases in Sobolev spaces in 1996 [28] and orthogonal multi-wavelets were constructed by Donovan et al. [29]. By $L_2(\mathbf{R})$, the linear space of all square-integrable real-valued functions is denoted on $\mathbf{R}$. The inner product in $L_2(\mathbf{R})$ is defined as

$$\langle u, v\rangle : = \int_R u(x)v(x)\mathrm{d}x, \quad u, v \in L^2(R)$$

If $\langle u,v \rangle = 0$, then u and v are regarded to be orthogonal. The norm of a function f in $L_2(\mathbf{R})$ is given by $\| f_2 \| : = \sqrt{\langle f, f\rangle}$. Let ϕ_1 and ϕ_2 be the cubic splines supported on interval $[-1, 1]$, they are given by

$$\phi_1(x) : = \begin{cases} (x+1)^2(1-2x) & \text{for } x \in [-1, 0] \\ (1-x)^2(1+2x) & \text{for } x \in [0, 1] \\ 0 & \text{for } x \notin [-1, 1] \end{cases} \tag{1}$$

and

$$\phi_2(x) : = \begin{cases} (x+1)^2 x & \text{for } x \in [-1, 0] \\ (x-1)^2 x & \text{for } x \in [0, 1] \\ 0 & \text{for } x \notin [-1, 1] \end{cases} \tag{2}$$

The graphs of ϕ_1 and ϕ_2 are depicted in Fig. 1. Clearly, both ϕ_1 and ϕ_2 belong to $C^1(\mathbf{R})$. The main reason for choosing these two spline functions to generate wavelets is that the corresponding wavelets would be equipped with the capability of orthogonal with respect to the inner product $\langle u', v' \rangle$ (i.e. $\langle u', v' \rangle = 0$), and this is the main integral term of the numerical method to analyze mechanical structure. Therefore, the multi-scale solution equation will be decoupled accordingly.

Heil et al. considered the possibility of construction of wavelets on the basis of Hermite cubic splines [30]. Dahmen et al. constructed biorthogonal multi-wavelets on the basis of the Hermite cubic splines ϕ_1 and ϕ_2 [31]. These wavelets were adapted to the interval [0, 1]. However, their construction for the wavelet basis on interval [0, 1] was quite complicated. Jia et al. constructed wavelet bases of Hermite cubic splines [26]. The corresponding wavelets ψ_1 and ψ_2 supported on interval $[-1, 1]$ are

$$\begin{cases} \psi_1(x) = -2\phi_1(2x+1) + 4\phi_1(2x) - 2\phi_1(2x-1) - 21\phi_2(2x+1) + 21\phi_2(2x-1) \\ \psi_2(x) = \phi_1(2x+1) - \phi_1(2x-1) + 9\phi_2(2x+1) + 12\phi_2(2x) + 9\phi_2(2x-1) \end{cases} \tag{3}$$

They satisfy the conditions $\langle \psi_1', \phi_m'(\bullet - j)\rangle = \langle \psi_2', \phi_m'(\bullet - j)\rangle = 0$, $m = 1, 2$, $\forall j \in Z$, where symbols $\bullet$ and j denote arbitrary variable and shift parameter, respectively. The shifts of ψ_1 and ψ_2 generate the wavelet space $\mathbf{W}$. Fig. 2 shows the graphic of ψ_1 and ψ_2. Obviously, ψ_1 is symmetric and ψ_2 is anti-symmetric.

The above-mentioned wavelets can generate a wavelet basis for the space $H_0^1(0, 1)$. Therefore, the following decomposition of $H_0^1(0, 1)$ are

$$H_0^1(0, 1) = V_1 \dotplus W_1 \dotplus W_2 \dotplus \cdots \tag{4}$$

where $\dotplus$ denotes direct sum, V_1 is the initial scaling space, and W_j ($j = 1, 2, \ldots$) is the wavelet space at the different level.

The scaling functions $\phi_{1,k}$ (Fig. 3) are

$$\begin{cases} \phi_{1,1}(x) : = \sqrt{\frac{5}{24}}\phi_1(2x-1) \\ \phi_{1,2}(x) : = \sqrt{\frac{15}{4}}\phi_2(2x) \\ \phi_{1,3}(x) : = \sqrt{\frac{15}{8}}\phi_2(2x-1) \\ \phi_{1,4}(x) : = \sqrt{\frac{15}{4}}\phi_2(2x-2) \end{cases} \tag{5}$$

Due to the boundary problem, Hermitian interpolation wavelet base cannot get transformation matrix, so it cannot be used as finite element interpolation function independently. A modified Hermitian scaling function is presented in this paper, so as to interpolate the field functions in wavelet finite element, and this modified Hermitian scaling function retains all kinds of the good performance of Hermitian wavelet.

The modified scaling functions $\phi_{1,k}'$ are

$$\begin{cases} \phi_{1,1}(x) : = \sqrt{\frac{5}{24}}\phi_1(2x) \\ \phi_{1,2}(x) : = \sqrt{\frac{5}{24}}\phi_1(2x-1) \\ \phi_{1,3}(x) : = \sqrt{\frac{15}{4}}\phi_2(2x) \\ \phi_{1,4}(x) : = \sqrt{\frac{15}{8}}\phi_2(2x-1) \\ \phi_{1,5}(x) : = \sqrt{\frac{15}{4}}\phi_2(2x-2) \\ \phi_{1,6}(x) : = \sqrt{\frac{5}{24}}\phi_1(2x-2) \end{cases} \tag{6}$$

and the wavelets $\psi_{j,k}$ are

$$\begin{cases} \psi_{j,k}(x) : = \frac{2^{-j/2}}{\sqrt{729.6}}\psi_1\left(2^j x - \frac{k}{2}\right) & \text{for } k = 2, 4, \ldots, 2^{j+1}-2 \\ \psi_{j,k}(x) : = \frac{2^{-j/2}}{\sqrt{153.6}}\psi_2\left(2^j x - \frac{k-1}{2}\right) & \text{for } k = 3, 5, \ldots, 2^{j+1}-1 \\ \psi_{j,1}(x) : = \frac{2^{-j/2}}{\sqrt{76.8}}\psi_2(2^j x) \\ \psi_{j,2^{j+1}}(x) : = \frac{2^{-j/2}}{\sqrt{76.8}}\psi_2(2^j x - 2^j) \end{cases} \tag{7}$$

All the modified scaling functions $\phi_{1,k}'$ and wavelets functions $\psi_{j,k}$ on interval [0, 1] are shown in Figs. 4 and 5. The special properties of wavelet bases of HCSWI are

$$\langle (\phi_{1,k}')', \psi_{j,k}' \rangle = \int_0^1 (\phi_{1,k}')' \psi_{j,k}' \,\mathrm{d}x = 0 \text{ for all } j \text{ and } k \tag{8}$$

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