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Interval finite element analysis and reliability-based optimization of coupled structural-acoustic system with uncertain parameters

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ABSTRACT

A modified interval parameter perturbation finite element method (MIPPM) and a reliability-based optimization model are proposed for the coupled structural-acoustic field prediction and structural design with uncertainties in both the physical parameters and boundary conditions. Interval variables are used to quantitatively describe all the uncertain parameters with limited information. The interval matrix and vector are expanded by the modified Taylor series. Compared with the traditional perturbation method, the proposed MIPPM can yield more accurate ranges of the uncertain structural-acoustic field, in which the higher order terms of Neumann series are employed to approximate the interval matrix inverse. The reliability idea is introduced to establish an interval optimization model relying on the satisfaction degree of interval. The uncertain constraints can be transformed into deterministic ones if given the confidence level. The proposed MIPPM is used to predict the intervals of the constraints, and whereby eliminate the optimization nesting. Numerical results about a 3D car are given to demonstrate the feasibility and efficiency of the proposed model and algorithm.

1. Introduction

In recent years, with the increase of people's awareness of environment and requirement for comfort, research on acoustic behavior inside the cabin is becoming a challenging problem for the lager complex structures such as cars, boats and airplanes. Therefore, predicting and improving the acoustic performance in terms of numerical simulation at the design stage has a very important significance. In many engineering situations, the noise induced by the coupling between the fluid and the structure is often encountered. The coupled structural-acoustic problem incorporates the mutual influence between the thin structure and acoustic field when the feedback of the acoustic fluid on the structure cannot be ignored [1]. The analytical solutions of the coupled structural-acoustic problem exist only for the simple shapes, while for the real-life problem, the numerical techniques become more practical, which can be grouped into two categories: deterministic approaches [2–4] and statistical analysis [5]. The deterministic approaches, such as the finite element method (FEM), boundary element method (BEM) and infinite element method (IEM), are concentrated towards the structural and acoustic response in the low-frequency range. In the high-frequency range, the response is very sensitive to small changes in the model, and thus the statistical energy analysis (SEA), introduced by Lyon and Maidanik in the 1960s [6], places great emphasis on the high-frequency response analysis by using time and space statistical information. In the last two decades, design of structural and acoustic systems that reduces noise and vibration through optimization has become very important. Hinton and Rao [7] applied shape optimization techniques to obtain optimal shell geometry and thickness distribution with the objective of a maximized fundamental frequency of structures. Marburg [8] described the noise transfer function including structural harmonic analysis, acoustic influence coefficients and their coupling, and proposed the well-known semianalytic calculation of harmonic displacement sensitivities. In the literature [9], a topology optimization based approach was proposed to study the optimal configuration of stiffeners for the interior sound reduction.

Traditional acoustic analysis has been conducted under the assumption that the physical properties and boundary conditions are deterministic. But in the actual engineering, due to the model inaccuracies, physical imperfections and system complexities, uncertainties in material properties, geometric dimensions and boundary conditions are unavoidable, which will lead to the uncertainty of the acoustic field. Popular approaches for these uncertain problems are probabilistic methods, where the

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probability density functions are defined unambiguously. Allen and Vlahopoulos [10] combined boundary element methods and finite element methods with stochastic analysis to calculate noise radiated from a structure subjected to random excitation. Chen et al. [11] developed the computing technique for the acoustic pressure spectral density and its sensitivity of coupled structural–acoustic systems subjected to stochastic excitation. James and Dowling [12,13] investigated the methods to quantify the uncertainties to determine the probability density functions of the sound pressure amplitude in the acoustic domain. For the probabilistic methods, a great amount of statistical information is required to construct the precise probability distribution functions of uncertain parameters. Unfortunately, for many engineering problems, it is often too difficult or costly to collect enough information about the uncertainty.

In order to overcome the shortcomings of probabilistic methods, some non-probabilistic approaches such as convex model and interval analysis have achieved widespread attention to deal with the uncertain problems without sufficient information [14,15]. Neumaier [16] investigated the hypercube approximation for the united solution set of the interval equations by the Gaussian elimination scheme, which will be extremely conservative due to a large number of elimination operations [17]. If the monotonicity of function was guaranteed, the exact response intervals can be obtained by the vertex method using all possible combinations of the interval parameters [18]. However, the computational effort of this method increases exponentially with the increase in the number of uncertain parameters. Compared with above interval approaches, the interval perturbation finite element method, proposed by Qiu et al. [19,20], has been widely applied in the structural response analysis thanks to its simplicity and efficiency [21]. On this basis, Xia and Yu developed the interval and subinterval perturbation methods to predict the acoustic field with uncertain-but-bounded parameters [22,23]. Up to now, research on interval uncertain problems is mainly concentrated in the structural field, while the application of interval approaches in the acoustic domain is still unexplored enough [24].

Hence, establishing effective numerical methods and optimization models for the structural-acoustic system with interval parameters has great significance both in theory and engineering. Firstly, finite element equations suitable to coupled structural-acoustic system are briefly presented. Subsequently in the next section, considering higher order terms, a modified parameter perturbation method is proposed to approximately predict the response intervals in the frequency domain. In Section 4, the new model and algorithm of interval reliability optimization are proposed based on the satisfaction degree of interval. A 3D car numerical example is given for verification in Section 5, and we conclude the paper with a brief discussion at last.

2. Structural-acoustic dynamic equilibrium equation

For a structural-acoustic system with the flexible boundary structure as shown in Fig. 1, in the frequency domain, the interior acoustic steady-state pressure p is governed by the Helmholtz equation

$$\nabla^2 p + \left(\frac{\omega}{c}\right)^2 p = 0 \tag{1}$$

where ∇^2 denotes the Laplace operator; ω is the angular frequency and c stands for the speed of sound.

For the interior acoustic field bounded by Γ, boundary conditions are usually grouped into three sets: the Dirichlet condition, Neumann condition and Robin condition.

$$p = p_w \ \ on \ \ \Gamma_D$$

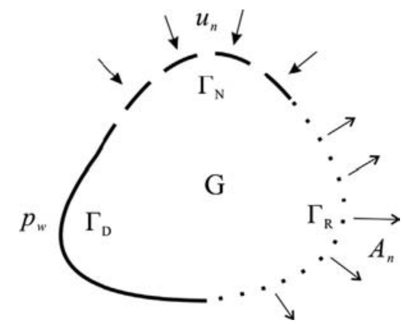

Fig. 1. Coupled structural-acoustic system.

$$\frac{\partial p}{\partial \mathbf{n}} = \rho\omega^2 u_n \ \ on \ \ \Gamma_N$$

$$\frac{\partial p}{\partial \mathbf{n}} = -j\rho\omega A_n p \ \ on \ \ \Gamma_R \tag{2}$$

where $\mathbf{n}$ is the exterior unit-normal vector; ρ is the density of fluid in acoustic field; u_n is the normal displacement of the wall structure on the Neumann boundary; A_n is the admittance coefficient that models the acoustic damping of the Robin boundary; $j = \sqrt{-1}$ is an imaginary unit.

Based on the work of Ohayon and Soize [25], we can obtain the following weak form by multiplying the Helmholtz equation with a weighting function δp

$$\int_{\Omega}\left(\nabla^2 p + \left(\frac{\omega}{c}\right)^2 p\right)\delta p d\Omega - \int_{\Gamma_N}\left(\frac{\partial p}{\partial \mathbf{n}} - \rho\omega^2 u_n\right)\delta p d\Gamma$$
$$-\int_{\Gamma_R}\left(\frac{\partial p}{\partial \mathbf{n}} + j\rho\omega A_n p\right)\delta p d\Gamma = 0 \tag{3}$$

In the standard finite element analysis, coupled structural-acoustic system is discretized to the isoparametric hexahedral elements. Based on the Lagrange interpolation shape functions, one can gain the following acoustic dynamic equilibrium equation

$$(\mathbf{K}^a + j\omega\mathbf{C}^a - \omega^2\mathbf{M}^a)\mathbf{P} = \rho\omega^2\mathbf{S}\mathbf{U}^s \tag{4}$$

where $\mathbf{K}^a$, $\mathbf{C}^a$, $\mathbf{M}^a$ and $\mathbf{S}$ stand for the global acoustic stiffness matrix, damping matrix, mass matrix and coupled matrix; $\mathbf{P}$ is the acoustic pressure vector and $\mathbf{U}^s$ is the displacement vector of the structure.

Similarly, considering the reaction on the structure made by interior acoustic field, the structural displacement can be predicted by calculating the finite element equation

$$(\mathbf{K}^s + j\omega\mathbf{C}^s - \omega^2\mathbf{M}^s)\mathbf{U}^s = \mathbf{F}^s + \mathbf{F}^a \tag{5}$$

where $\mathbf{K}^s$, $\mathbf{C}^s$ and $\mathbf{M}^s$ represent the structural system stiffness matrix, damping matrix and mass matrix, respectively; $\mathbf{F}^s$ denotes the external load vector applied to the structure, while $\mathbf{F}^a$ stands for the generalized load vector made by the acoustic pressure.

Based on the principle of virtual work, the equivalent force vector made by interior acoustic field on the structure can be expressed as

$$\mathbf{F}^a = \mathbf{S}^T\mathbf{P} \tag{6}$$

Combining Eq. (4) with Eq. (5), the dynamic equilibrium equation of the coupled structural-acoustic system can be denoted as

$$-\omega^2\mathbf{M}\mathbf{U} + j\omega\mathbf{C}\mathbf{U} + \mathbf{K}\mathbf{U} = \mathbf{F} \tag{7}$$