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Review

Wavelet-based numerical analysis: A review and classification

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ABSTRACT

Wavelet analysis is a new method called 'numerical microscope' in signal and image processing. It has the desirable advantages of multi-resolution properties and various basis functions, which fulfill an enormous potential for solving partial differential equations (PDEs). The numerical analysis with wavelet received its first attention in 1992, since then researchers have shown growing interest in it. Various methods including wavelet weighted residual method (WWRM), wavelet finite element method (WFEM), wavelet boundary method (WBM), wavelet meshless method (WMM) and wavelet-optimized finite difference method (WOFD), etc. have acquired an important role in recent years. This paper aims to make a comprehensive review and classification on wavelet-based numerical analysis and to note their merits, drawbacks, and future directions. And thus the present review helps readers identify research starting points in wavelet-based numerical analysis and guides researchers and practitioners.

Contents

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1. Introduction

In numerical analysis, classical discretization methods, such as finite differences, finite elements, spectral elements, are powerful tools for solving PDEs. However, singularities and steep changes often emerge in many phenomena, like stress concentration, elastoplasticity, shock wave and crack. Since small-scale features only exist in a small percentage of the solution domain, if one chooses a uniform numerical grid fine enough to resolve the small-scale characteristics, then the solution to the equations will be over-resolved in the majority of the domain. One would like, ideally, to have a dense grid where small-scale structure exists and a sparse grid where the solution is only composed of large-scale features [1]. It demands for the usage of non-uniform grids or moving elements to dynamically adapt to the changes in the solution. That is where wavelets play a role.

Wavelet is called "numerical microscrope" in signal and image processing. It has been 31 years since Morlet proposed the concept of wavelet analysis to automatically reach the best trade-off between time and frequency resolution [2]. Later, this proposition was considered as a generalization of ideas promoted by Haar (1910), Gabor (1946) [3], and Zweig (1976) [4]. Wavelet was in the air in the numerical analysis community in the early 1990s [5]. Generally, wavelet is used to describe a function that features compact support, i.e. it is nonzero only on a finite interval. The representation of a set of time-dependent data on a wavelet basis leads to a unique structure of information that is localized simultaneously in the frequency and time domains. This does not occur in a Fourier representation, where specific frequencies cannot be associated with a particular time interval, since the basis functions have constant resolution on the entire domain. A wavelet basis representation originates a set of wavelet coefficients structured over different levels of resolution. Each coefficient is associated with a resolution level and a point in the time domain. The coefficients involved in the lowest-resolution level describe the low-frequency features of the data spanning over broad time intervals. At the highest level, the coefficients are associated with highly localized high-frequency features [6]. These desirable advantages draw sight of researchers to apply wavelets in the resolution of PDEs [7–13]. In the case of a moving steep front, using the wavelet transformation one can track its position and increase the local resolution of the grid by adding higher resolution wavelets in that region. On the other hand, the resolution level in the smoother regions can be appropriately decreased, avoiding an unnecessarily dense grid.

In 1991, Beylkin firstly carried out the study of numerical calculation based on wavelet. The study was presented in the form of conference paper [14], military AD report [15] and official journals [16,17]. Daubechies wavelet was used in the calculation process. Subsequently, Jaffard proved the superiority of solving elliptic partial differential equations by use of wavelet [18,19]. Zweig found that the generalized wave equation, on which the continuous wavelet transform is based, can be used to understand phenomena related to the hearing process [20]. Dahmen and Chen initiated related studies [21,22]. These early research literatures had a great effect and motivated the applications of wavelet in numerical calculation. Hereafter, a lot of research institutions and universities began to conduct the study of wavelet. These papers showed that wavelet multi-resolution and wavelet properties, including compact support, vanishing moment and norm equivalence, were superior and universal in equation solution.

Bertoluzza et al. studied error estimation and convergence of wavelet collocation methods and wavelet numerical algorithms [23–26]. Cohen and Kaber et al. studied wavelet finite volume methods and wavelet multiscale algorithms [27,28]. Berrone and Dmmel et al. studied wavelet-Galerkin method in any solving domain [29]. Radha, Williams and Amaratunga applied wavelet finite element to the study of microscale molecular structure [30,31]. Chen and Micchelli studied Galerkin methods of discrete wavelet [32]. Sweldens and Piessens et al. performed a series of wavelet application studies in numerical analysis [33]. In addition, Chen, Li and He et al. have successively done studies on wavelet based error estimator and adaptive schemes [34–37].

During the past two decades, the theories of wavelet numerical methods have been developed in a variety of directions. In summary, from the aspect of algorithm construction, the main wavelet-based numerical analysis methods are categorized as follows:

- Wavelet weighted residual method (WWRM);
- Wavelet finite element method (WFEM);
- Wavelet boundary element (WBE);
- Wavelet meshless method (WMM)
- Other wavelet-based numerical methods

Among the above-mentioned methods, weighted residual method is the earliest numerical method in computational mechanics, so it is necessary to illustrate in an independent class although it has many common ground with other methods. Based on the Galerkin method, the famous weighted residual method, finite element method (FEM) is proposed. FEM is more universal compared with other methods. Many successful software such as ANSYS, NASTRAN are constructed on the basis of FEM. However, the accuracy of FEM is not the best in certain fields. To improve the accuracy and efficiency of FEM, the boundary element method (BEM) is proposed. Furthermore, meshing is an important and tough task in FEM or BEM for some complex shapes. To ease this problem, the meshless method is introduced. In the classical numerical methods, in order to approximate unknown functions, it is key and necessary to construct the so-called shape functions, which are complicated, time consuming and even hard to realize in some special conditions. In addition, the complexity of shape functions will result in the increase of computational cost in the total solution process. It is desirable to find a new method which is simple and reasonable to construct shape functions. However, it seems to be a complicated task. So we should resort to some other mathematics tools. Combined with wavelet, the classical methods show a new appearance in performance. They act as the basic foundation and theoretical background of classification.