Finite Elements in Analysis and Design 81 (2014) 48–56

Contents lists available at ScienceDirect

Finite Elements in Analysis and Design

journal homepage: www.elsevier.com/locate/finel

A finite element model to simulate long-term behavior of prestressed concrete girders

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ARTICLE INFO

Article history:
Received 21 August 2013
Received in revised form
29 October 2013
Accepted 25 November 2013
Available online 25 December 2013

Keywords:
Creep
Finite element
Prestressed concrete
Relaxation
Time-dependent analysis

ABSTRACT

The development of a finite element model for time-dependent analysis of bonded prestressed concrete girders at service conditions is presented. The effects of creep and shrinkage of concrete and relaxation of steel tendons are taken into account. The concrete creep is modeled based on the Dirichlet series creep compliance with efficiency in simulating the stress history. In addition, the interaction between different time-dependent effects is fully considered in the numerical procedure. The numerical method is formulated based on the layered Euler–Bernoulli beam theory. In the constructed incremental equilibrium equations, the equivalent nodal load increments consist of four components contributed by external loads, concrete creep, concrete shrinkage and tendon relaxation, while the stiffness matrix is composed of the material and geometric stiffness matrices. Numerical examples show that the proposed model can well predict the long-term behavior of prestressed concrete beams, and that the time-dependent effects have important influence on the structural behavior.

1. Introduction

For a prestressed concrete structure under sustained loads, the section stress and strain are subject to change with time due to time-dependent effects resulting from creep and shrinkage of concrete and relaxation of prestressing tendons [1]. If the structure is statically indeterminate, the time-dependent effects will also result in redistribution of moments in addition to the stress redistribution in cross sections [2]. The inevitable loss of the long-term workability and performance of prestressed concrete structures at service conditions is a primary concern for researchers and engineers.

The theoretical simulation of long-term behavior of prestressed concrete structures is not an easy task. One of the challenges is to model accurately and efficiently the creep effect, since creep of concrete is associated with the history of the applied stress. A number of methods have been developed to simulate the creep behavior of concrete members [3,4]. Utilization of the effective modulus is known to be the simplest method for a creep analysis [5]. This method may produce satisfactory results when the aging effects are negligible, but it will lead to an overestimation of the creep when the aging effects are present. Most of the available models were based on the age-adjusted effective modulus method (AEMM) by introducing an aging coefficient in the effective modulus [6–8]. The values of the aging coefficient are often taken from tables or charts and, therefore, AEMM is a simplified method rather than a refined method. Another challenge is to model accurately the relaxation of prestressing tendons, taking into account the interaction between different time-dependent effects. Commonly, this interaction is considered approximately by using a relaxation reduction coefficient obtained from tables or charts [1,9]. Although several refined numerical methods have recently been developed to model the concrete creep and/or tendon relaxation [10,11], few of these methods have been applied to continuous concrete members.

This paper presents a rigorous numerical method for time-dependent analysis of bonded prestressed concrete girders at service loads, taking into account concrete creep, concrete shrinkage and steel tendon relaxation. The finite element method is formulated based on the layered Euler–Bernoulli beam theory combined with rational models for concrete creep and tendon relaxation. The proposed method of analysis can be applied to the prediction of long-term behavior of both simply support and continuous girders. Several numerical examples are given to illustrate the reliability and applicability of the proposed model.

2. Material models

2.1. Instantaneous constitutive laws

Since this study concerns the time-dependent analysis of prestressed concrete girders at service loads, the following simplifications of the instantaneous constitutive laws of materials may be made.

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http://dx.doi.org/10.1016/j.finel.2013.11.007

The concrete in compression at service conditions is assumed to be within the elastic range. The concrete in tension is assumed to be linear elastic prior to cracking, followed by linear tension stiffening behavior, as indicated by

$$\sigma_c = E_c \varepsilon_c \quad for\ \varepsilon_c \le \varepsilon_{cr} \tag{1a}$$

$$\sigma_c = f_t \left[1 - \frac{\varepsilon_c - \varepsilon_{cr}}{\varepsilon_{tu} - \varepsilon_{cr}}\right] \quad for\ \varepsilon_{cr} < \varepsilon_c \le \varepsilon_{tu} \tag{1b}$$

where f_t is the concrete tensile strength; E_c is the modulus of elasticity of concrete; $\varepsilon_{cr} = f_t/E_c$; and ε_{tu} is the ultimate concrete tensile strain. When the concrete strain is greater than ε_{tu}, the tensile stress is equal to zero. Both the reinforcing and prestressing steel may be considered to be linear elastic at service conditions.

2.2. Concrete creep model

At service conditions, creep can be considered to be proportional to the applied stress. Therefore, the creep strain $\varepsilon_c^{cr}(t)$ under sustained stress is given by

$$\varepsilon_c^{cr}(t) = \sigma_c(\tau) C(t, \tau) \tag{2}$$

where $\sigma_c(\tau)$ is the concrete stress at age τ; and $C(t,\tau)$ is the creep compliance, defined as the creep strain at age t caused by a unit stress applied at age τ.

When the applied stress is subject to a gradual change with time, which is generally true in practical applications, the creep strain due to the applied stress is calculated by applying the principle of superposition as follows:

$$\varepsilon_c^{cr}(t) = \sigma_c(t_0) C(t, t_0) + \int_{t_0}^{t} C(t, \tau) \frac{\partial \sigma_c(\tau)}{\partial \tau} d\tau \tag{3}$$

where t_0 is the age at which the initial stress is applied. This equation indicates that creep is associated with the history of the applied stress.

The time is divided into a number of small intervals to apply an incremental method. Generally, the stresses for each element at various time have to be stored for integrating Eq. (3). However, this may have some limitations in application to large structures. To overcome this problem, the following Dirichlet series creep compliance, initially proposed by Zienkiewicz and Watson [12], is adopted in the present study:

$$C(t, \tau) = \sum_{k=1}^{m} \phi_k(\tau)[1 - e^{-r_k(t-\tau)}] \tag{4}$$

where m, $\phi_k(\tau)$ and r_k are the empirical parameters to be determined by experimental data.

By utilizing the above creep compliance, the creep strain increment at time interval Δt_n $(= t_n - t_{n-1})$, $\Delta\varepsilon_c^{cr}$, is given by [13]

$$\Delta\varepsilon_c^{cr} = \varepsilon_c^{cr}(t_n) - \varepsilon_c^{cr}(t_{n-1}) = \sum_{k=1}^{m} (1 - e^{-r_k \Delta t_n}) \omega_{kn} + C(t_n, t_{n-1/2}) \Delta\sigma_n \tag{5}$$

in which $t_{n-1/2}$ represents the middle time between time t_{n-1} and time t_n; $\Delta\sigma_n$ is the stress increment at time interval Δt_n; ω_{kn} is obtained from the following recursive formula:

$$\omega_{kn} = \omega_{k(n-1)} e^{-r_k \Delta t_{n-1}} + \Delta\sigma_{n-1} \phi_k(t_{(n-1)-1/2}) e^{-r_k \Delta t_{n-1/2}} \tag{6a}$$

$$\omega_{k1} = \sigma_c(t_0) \phi_k(t_0) \tag{6b}$$

From Eqs. (5) and (6), it can be seen that, instead of recording the entire stress history, only the value of $\omega_{k(n-1)}$ needs to be stored, thus ensuring an efficient creep analysis.

2.3. Concrete shrinkage model

Shrinkage of concrete is defined as the volume change which is independent of the imposed stress. Therefore, the shrinkage strain can be conveniently calculated using the available shrinkage models proposed by the codes or by the investigators. According to Mode Code 2010 [14], the shrinkage strain is calculated from

$$\varepsilon_c^{sh}(t) = \varepsilon_{cas}(t) + \varepsilon_{cds}(t - t_s) \tag{7}$$

in which $\varepsilon_{cas}(t)$ and $\varepsilon_{cds}(t - t_s)$ are the autogenous shrinkage and the drying shrinkage, respectively. They are expressed as follows:

$$\varepsilon_{cas}(t) = \varepsilon_{cas0} \beta_{as}(t) \tag{8a}$$

$$\varepsilon_{cds}(t - t_s) = \varepsilon_{cds0} \beta_{RH} \beta_{ds}(t - t_s) \tag{8b}$$

$$\varepsilon_{cas0} = -\alpha_{as} \left(\frac{f_{cm}/10}{6 + f_{cm}/10}\right)^{2.5} \times 10^{-6} \tag{9a}$$

$$\beta_{as}(t) = 1 - e^{-0.2\sqrt{t}} \tag{9b}$$

$$\varepsilon_{cds0} = [(220 + 110\alpha_{ds1}) e^{-\alpha_{ds2} f_{cm}}] \times 10^{-6} \tag{9c}$$

$$\beta_{ds}(t - t_s) = \left(\frac{t - t_s}{0.035h^2 + (t - t_s)}\right)^{0.5} \tag{9d}$$

where α_{ds1} and α_{ds2} are the coefficients depending on the type of cement; f_{cm} is the mean concrete compressive strength at the age of 28 days; β_{RH} is the coefficient depending on the relative humidity of the ambient atmosphere; h is the notional size of member; $(t–t_s)$ is the duration of drying.

The shrinkage strain increment, $\Delta\varepsilon_c^{sh}$, at time interval Δt_n is given by

$$\begin{aligned} \Delta\varepsilon_c^{sh} &= \varepsilon_c^{sh}(t_n) - \varepsilon_c^{sh}(t_{n-1}) \\ &= \varepsilon_{cas0}[\beta_{as}(t_n) - \beta_{as}(t_{n-1})] + \varepsilon_{cds0} \beta_{RH} [\beta_{ds}(t_n - t_s) - \beta_{ds}(t_{n-1} - t_s)] \end{aligned} \tag{10}$$

2.4. Relaxation model for steel tendons

In this study, the relaxation of prestressing steel, σ_{pr}, is evaluated utilizing the equation proposed by Magura et al. [15]:

$$\frac{\sigma_{pr}}{\sigma_{p0}} = -\frac{\log(\tau - t_0)}{10} \left(\frac{\sigma_{p0}}{f_{py}} - 0.55\right) \tag{11}$$

in which $(\tau–t_0)$ is the time in hours after stressing; σ_{p0} is the initial stress immediately after stressing; and f_{py} is the yield stress of prestressing steel. The ratio of the yield stress to the ultimate tensile strength generally varies between 0.8 and 0.9, depending on the type of the prestressing steel.

It should be noted that the above relaxation equation is subject to the condition that the tendon length is kept constant and σ_{p0} is the only applied stress. In prestressed concrete members, however, the applied stress would be influenced by some causes such as the prestress transfer, the load application, and the interaction between the creep and shrinkage of concrete and the relaxation of steel tendons. Therefore, the initial stress for computing the stress relaxation at each time interval should be appropriately adjusted according to the change of the tendon stress as a result of these causes. The procedure for computing the actual relaxation of prestressing steel is illustrated in Fig. 1. Denote by $\sigma_{p0(1)}$ the initial prestress at time t_0. At time t_1, the prestress varies from $\sigma_{p0(1)}$ to σ_{p1} due to the tendon relaxation $\Delta\sigma_{pr1}$ and also to other causes. Compute the fictitious initial prestress $\sigma_{p0(2)}$ using Eq. (11) such that $\sigma_{p0(2)}$ would be relaxed to σ_{p1} from time t_0 to time t_1. Based on the fictitious initial prestress $\sigma_{p0(2)}$, the tendon relaxation $\Delta\sigma_{pr2}$