Finite Elements in Analysis and Design 80 (2014) 33–40

Contents lists available at ScienceDirect

Finite Elements in Analysis and Design

journal homepage: www.elsevier.com/locate/finel

Adaptive response surface based efficient Finite Element Model Updating

Subrata Chakraborty *, Arunabh Sen

Department of Civil Engineering, Bengal Engineering and Science University, Shibpur, Howrah, India

ARTICLE INFO

Article history:
Received 20 June 2013
Received in revised form
4 November 2013
Accepted 6 November 2013
Available online 6 December 2013

Keywords:
Finite Element Model Updating
Response surface
Moving least-squares method

ABSTRACT

The requirement of repeated evaluation of structural responses in typical sensitivity based Finite Element Model Updating (FEMU) procedure limits its popular applications for large structures. The least-squares method (LSM) based response surface method (RSM) is applied as a potential alternative for responses approximations in iterative model updating procedure. However, the LSM is a major source of error in response prediction and the moving least-squares method (MLSM) is found to be more efficient in this regard. An attempt has been made in the present study to explore the effectiveness of MLSM based RSM in FEMU. A comparative assessment is performed between the MLSM based and the conventional LSM based RSM for model parameter updating. The comparative study is being illustrated with the help of two example problems using artificially generated input responses. It is generally observed that the MLSM based RSM identifies better than the LSM based approach.

1. Introduction

The Finite Element Model Updating (FEMU) has emerged as a subject of immense importance and are used in many applications such as damage detection, structural health monitoring (SHM), structural control, structural evaluation and assessment [1,2]. A number of FEMU procedures providing clear overview of structural model updating may be seen in [3,4]. The non-iterative methods that directly update the finite element (FE) model properties are one-step procedures [5,6]. The iterative FEMU procedure uses the sensitivities of the parameters to find their changes involving solution of an optimization problem. There are examples of successful application of FEMU strategy using both static displacements and strains [7–9] obtained by less complicated measurement and data processing than those for dynamic ones. It was observed that the static response based FEMU strategy under moderate noise level show higher effectiveness and accuracy if more response information is provided. A recent baseline FE model for bridge management and calibration using non-destructive test (NDT) data is notable in this regard [10]. It is noted that the mathematical model used in the model updating is usually ill posed and the special attention is required for an accurate solution [11]. Jaishi and Ren [12,13] used either single-objective or multi-objective optimization technique to update the FE models of structures.

The aforementioned traditional sensitivity-based FEMU methods require complicated constructions of sensitivity matrices because the FE models should be tuned and recomputed during optimization process. For a large FE model repeated evaluation of performance function and local gradients are not only computational intensive, but may also result convergence difficulty, especially when complex nonlinear constitutive behaviors are involved. Thus, for a large FE model requiring update of huge geometric and physical structural parameters can rule out many FEMU approaches. Furthermore, the mechanical model and the iterative updating method need to be merged together for complex problem. For this, the structural FE models are often constructed by using various commercial FE softwares like ANSYS, ABAQUS and SAP etc. For this, each iteration needs to go back to run the FE analysis package with any parameter updated, which limits the popular application of FEMU procedure in practice. Hence, alternative techniques for efficient computation of response of large structures by overcoming the aforementioned drawbacks while retaining the accuracy is of paramount importance for FEMU of large structural system. Response Surface Method (RSM) based meta-modelling has emerged as a convenient alternative solution to such problems to achieve a balance. The primary advantages of RSM in the context of FEMU are its easy implementation and high cost-efficiency. Iwasaki et al. [14] introduced the idea of pursuing damage detection by comparing the actual response surface (RS) built on some monitored response variables with a reference RS associated with the undamaged state. Faravelli and Casciati [15] overviewed the recent developments and potential of RSM in SHM. Guo and Zhang [16] applied the central composite design to construct RS models for updating structural stiffness of an H-shaped structure

* Corresponding author. Tel.: +91 33 26684561; fax: +91 33 26682916.
E-mail addresses: schak@civil.becs.ac.in, schakbec@gmail.com (S. Chakraborty).

0168-874X/$ - see front matter © 2013 Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.finel.2013.11.002

using modal frequencies as the input response. The potential of RSM-based model updating has been explored by Fang and Perera [17] to identify the existing damage of a tested reinforced concrete frame and an experimental full-scale bridge. The RSM based FEMU procedure using uniform design to estimate the structural parameters based on measured natural frequencies [18] and static responses [19] are presented. It is generally observed that the RSM-based model updating is much more cost-effective than the traditional sensitivity-based FEMU procedure considering the likewise accurate predictions.

The applications of RSM in FEMU as discussed in the above are based on the global approximation of scatter position data, obtained by using the least-squares method (LSM). However, the LSM is one of the major sources of error in prediction by the RSM. The moving least square method (MLSM), basically a local approximation approach is found to be more efficient in this regard [20–22]. In the present study, an attempt has been made to explore the applicability and effectiveness of MLSM based RSM for FEMU using static responses. The methodology of updating procedure involves the selection of model parameters, evaluation of responses by FE analysis at judiciously selected set of input model parameters to create the RS model. A design of experiment (DOE) technique is utilized for selecting the efficient set of input model parameters considering their feasible range of variation. The iteration during the optimization process necessary for FEMU is carried out within the established RS model. However, during the iteration, the evaluation of structural response using RS based metamodel is performed by the MLSM based RSM. A comparative study of parameter identification is being made to study the superiority of the MLSM based approach over LSM approach. The comparative study is being illustrated with the help of two numerical examples using artificially generated input responses.

2. Response surface based Finite Element Model Updating

The RSM is a set of mathematical and statistical techniques designed to gain a better understanding about the overall response by DOE and subsequent analysis of experimental data. The method primarily uncovers analytically complicated or an unknown relationship between several inputs and desired output through empirical models (non-mechanistic) in which the response function is replaced by a simple function (often polynomial) that is fitted to data at a set of carefully selected points (referred as DOE), normally obtained from experimental investigation or numerical simulation. To create a response surface that will serve as a surrogate for the FE simulation model, the basic process consists of calculating predicted values of the response features at various sample points in the parameter space by performing an experiment at each of those points.

2.1. LSM based RSM

If there are n response values y_i corresponding to n numbers of observed data, x_{ij} (denotes the ith observation of the input variable x_j in a DOE), the relationship between the response and the input variables can be expressed by the following:

$$\mathbf{y} = X\boldsymbol{\beta} + \varepsilon_y \tag{1}$$

In the above multiple non-linear regression model $\mathbf{X}$, $\mathbf{y}$, $\boldsymbol{\beta}$ and ε_y are the design matrix containing the input data from the DOE, the response vector, the unknown co-efficient vector and the error vector, respectively. Typically, the quadratic polynomial form used in the RSM is as following:

$$\mathbf{y} = \beta_0 + \sum_{i=1}^{k} \beta_i x_i + \sum_{i1}^{k} \sum_{j=1}^{k} \beta_{ij} x_i x_j \tag{2}$$

The LSM of estimation technique is usually applied to obtain the unknown polynomial coefficient by minimizing the error norm defined as

$$L = \sum_{i=1}^{n} \left(y_i - \beta_0 - \sum_{i=1}^{k} \beta_i x_i - \sum_{i=1}^{k} \sum_{j=1}^{k} \beta_{ij} x_i x_j \right)^2 = (\mathbf{y} - \mathbf{X}\beta)^T (\mathbf{y} - \mathbf{X}\beta) \tag{3}$$

and the least squares estimate of β is obtained as

$$\beta = [\mathbf{X}^T \mathbf{X}]^{-1} \{\mathbf{X}^T \mathbf{y}\} \tag{4}$$

Once the polynomial coefficients β are obtained from the above equation, the response $\mathbf{y}$ can be readily evaluated for any set of input parameters. To fit an accurate model within reasonable time, it is required that the initial input data ($\mathbf{X}$ and $\mathbf{y}$) are selected judiciously. It may be realized that the accuracy of evaluation of structural parameters need to be updated will depend on the capability of a metamodel to capture the nonlinearity and the local variations of response behaviors. This is due to the fact that the accuracy of the updated parameters relies on how accurate the metamodel is in capturing the response variations during each iteration cycle of the numerical optimization procedure [23]. It may be noted that the construction of RS model as discussed above is the global approximation of scatter position data, obtained by using the LSM [24,25]. The MLSM based adaptive RSM; a local approximation procedure is found to be much more effective in evaluating RS polynomial accurately [20,25]. It is expected that the updated parameter will be more accurate and the number of iteration in the update procedure will be reduced if MLSM is used in lieu of LSM based RSM in the FEMU procedure. Keeping this in view, a new algorithm to obtain an improved RSM for FEMU in the framework of MLSM method is proposed.

2.2. MLSM based RSM

The MLSM based RSM is a weighted LSM that has varying weight functions with respect to the position of approximation. The weight associated with a particular sampling point x_i decays as the prediction point x moves away from x_i. The weight function is defined around the prediction point x and its magnitude changes with x. The least-squares function $L_y(x)$ can be defined as the sum of the weighted errors as following:

$$L_y(x) = \sum_{i=1}^{n} w_i \varepsilon_i^2 = \boldsymbol{\varepsilon}^T \mathbf{W}(x) \boldsymbol{\varepsilon} = (\mathbf{y} - \mathbf{x}\boldsymbol{\beta})^T \mathbf{W}(x) (\mathbf{y} - \mathbf{x}\boldsymbol{\beta}) \tag{5}$$

where, $\mathbf{W}(x)$ is the diagonal matrix of the weight function. It can be obtained by utilizing the weighting function such as constant, linear, quadratic, higher order polynomials, exponential functions, etc. [20,26]. In the present study, following exponential form is considered:

$$\begin{aligned} w(x - x_i) = w(d) &= \exp(-d/R_I) \quad && \text{if } d/R_I \le 1.0 \\ &= 0.0 && \text{if } d/R_I \ge 1.0 \end{aligned} \tag{6}$$

In the above, d is the distance of the point where approximate response is required to the origin of the approximating domain and R_I is the radius of the sphere of influence, chosen as twice the distance between the center point and the extreme most experimental point. The value of R_I is so chosen in order to secure sufficient number of neighboring experimental points so as to avoid singularity. More details about the calculation of R_I can be found elsewhere [20,27]. Eventually, a weight matrix $\mathbf{W}(x)$ can be constructed by using the weighting function in the diagonal terms