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A meshfree-based local Galerkin method with condensation of degree of freedom

Dean Hu [a,b,*], Yigang Wang [a], Yangyang Li [a], Yuantong Gu [b], Xu Han [a]

[a] State Key Laboratory of Advanced Design and Manufacturing for Vehicle Body, Hunan University, Changsha 410082, PR China
[b] School of Chemistry, Physics and Mechanical Engineering, Queensland University of Technology, GPO Box 2434, Brisbane, QLD 4001, Australia

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ABSTRACT

Condensation technique of degree of freedom is firstly proposed to improve the computational efficiency of meshfree method with Galerkin weak form. In present method, scattered nodes without connectivity are divided into several subsets by cells with arbitrary shape. The local discrete equations are established over each cell by using moving kriging interpolation, in which the nodes that located in the cell are used for approximation. Then, the condensation technique can be introduced into the local discrete equations by transferring equations of inner nodes to equations of boundary nodes based on cell. In the scheme of present method, the calculation of each cell is carried out by meshfree method with Galerkin weak form, and local search is implemented in interpolation. Numerical examples show that the present method has high computational efficiency and convergence, and good accuracy is also obtained.

1. Introduction

In the recent two decades, the development and application of meshfree methods have attracted much attention. One of the reasons is the versatility of meshfree methods for complex geometry of solids and flexibility for different engineering problems [1]. Element free Galerkin (EFG) method, which is originated by Belytschko et al. [2–5], is one of the most widely used meshfree methods. The key advantage of EFG method is that only nodal data is required and no element connectivity is needed, when moving least squares (MLS) interpolation is used to construct trial and test functions. However, because shape functions constructed by MLS interpolation do not possess Kronecker delta function property, the treatment of essential boundary conditions is one of the typical drawbacks. Thus, many special techniques have been proposed to impose essential boundary conditions [6–8], such as point collocation [6], Lagrange multipliers [2], singular weighting functions [7] and penalty method [8]. None of these methods is fully satisfactory, as they still need additional efforts to enforce essential boundary conditions.

In order to totally eliminate the drawback associated with EFG method for imposing essential boundary conditions, Liu and Gu have developed the Point Interpolation Methods (PIM) by using polynomial basis or/and radial basis function (RBF) [9–12]. Gu has firstly introduced the moving kriging (MK) interpolation-based meshfree method for solving a simple steady-state heat conduction problem [13]. Through PIM and MK interpolation, the shape functions with the delta function property can be obtained, and then the essential boundary conditions can be imposed easily. A comparison between the radial point interpolation method (RPIM) and the kriging interpolations is presented by Dai et al. for elasticity [14]. Moreover, Lam et al. has introduced a local kriging (LoKriging) method into two-dimensional (2D) solid mechanics problems [15], and Li et al. has further developed the LoKriging method for structural dynamics analysis [16]. Furthermore, a moving kriging interpolation-based element free Galerkin method is developed by Bui et al. for static analysis, structural dynamic analysis and free vibration analysis of Kirchhoff plates [17–19].

On the other hand, high computational cost is still one of main drawbacks in meshfree method with Galerkin weak form. The global search is implemented in approximation functions of displacement, even though the PIM or MK interpolation is used in the discrete formulations based on global or local Galerkin weak form. One of the reasons is the low efficiency of global search. In this paper, a local search technique is proposed based on cell, and condensation technique is used to improve computational efficiency of meshfree method with Galerkin weak form. Condensation technique is used to reduce the degree of freedom by transferring displacements of inner nodes to displacements of boundary nodes in system equations, which is usually used in finite element method (FEM) [20–22].

* Correspondence to: Hunan University, HNU College of Mechanical and Vehicle Engineering, State Key Laboratory of Advanced Design and Manufacturing for Vehicle Body, Changsha 410082, PR China. Tel.: +86 731 88822325.
E-mail address: hudean@hnu.edu.cn (D. Hu).

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The outline of this paper is as following. The discrete domain with distributed nodes and cells are described in Section 2. In Section 3, a brief summary of MK interpolation is given. In Section 4, we present the elastic static formulations of meshfree-based local Galerkin method, the condensation technique and the flowchart of present method. In Section 5, numerical examples are investigated and discussed. Finally, conclusions are given in Section 6.

2. Discrete domain of problem

Considering a problem with domain Ω and boundary Γ, as shown in Fig. 1 for 2D problems, scattered nodes without connectivity are divided into several subsets by cells with arbitrary shape. If a cell Ω_i intersects with an adjacent cell Ω_j, they connect with each other at intersection line Γ_{li} without overlapping. The union of all cells can cover the problem domain Ω, which yields

$$\Omega = \cup_{i=1}^{M}\Omega_i, \quad i=1,2\ldots M \tag{1}$$

and Γ_i is the boundary of cell Ω_i. Γ_i is combined by

$$\Gamma_i = \Gamma_{li} \cup \Gamma_{ti} \cup \Gamma_{ui} \tag{2}$$

where M is the total number of cells in the problem domain. Γ_{ti} and Γ_{ui} is the natural boundary and essential boundary of cell Ω_i, respectively. And Γ_{li} is the intersection line of neighboring cells. Scattered nodes at boundary Γ_i are referred to as boundary nodes, and nodes in cell Ω_i are referred to as inner nodes, as shown in Fig. 1.

In this paper, the local discrete governing equations are established based on scattered nodes in each cell by using MK interpolation, then the global discrete governing equations can be obtained by assembling all local discrete governing equations.

3. Moving kriging interpolation [13]

A cell Ω_i of neighborhood of point $\mathbf{x}$ is considered in the problem domain Ω. MK interpolation for approximation of field variable $\mathbf{u}$ can be defined as

$$\mathbf{u}^h(\mathbf{x}) = [\mathbf{p}^{\mathbf{T}}(\mathbf{x})\mathbf{A} + \mathbf{r}^{\mathbf{T}}(\mathbf{x})\mathbf{B}]\mathbf{u} = \sum_{I=1}^{n}\phi_I(\mathbf{x})u_I = \mathbf{\Phi}(\mathbf{x})\mathbf{u} \tag{3}$$

where n is the total number of nodes for interpolation. $\phi_I(\mathbf{x})$ is the MK shape functions

$$\phi_I(\mathbf{x}) = \sum_{j=1}^{m} p_j(\mathbf{x})A_{jI} + \sum_{k=1}^{n} r_k(\mathbf{x})B_{kI} \tag{4}$$

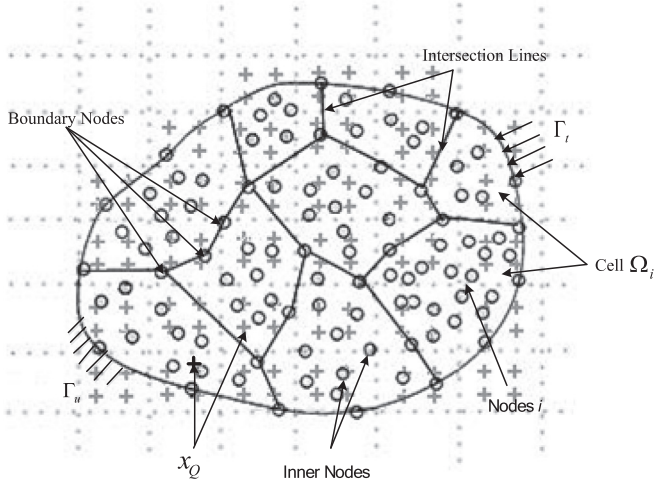

Fig. 1. Discrete domain of problem.

in which A_{jI} is the (j, I) element of matrix $\mathbf{A}$, and B_{kI} is the (k, I) element of matrix $\mathbf{B}$. Matrixes $\mathbf{A}$ and $\mathbf{B}$ can be written as following

$$\mathbf{A} = (\mathbf{P}^T\mathbf{R}^{-1}\mathbf{P})^{-1}\mathbf{P}^T\mathbf{R}^{-1} \tag{5a}$$

$$\mathbf{B} = \mathbf{R}^{-1}(\mathbf{I} - \mathbf{PA}) \tag{5b}$$

where $\mathbf{I}$ is a unit matrix. $\mathbf{P}$ is an $n \times m$ matrix and represents the collected values of vector $\mathbf{p}(\mathbf{x})$ at the neighboring interpolated nodes of $\mathbf{x}$. Vector $\mathbf{p}(\mathbf{x})$ is the polynomial with m basis functions

$$\mathbf{p}(\mathbf{x}) = \{p_1(\mathbf{x})\ p_2(\mathbf{x}) \cdots p_m(\mathbf{x})\}^T \tag{6}$$

and

$$\mathbf{P} = \begin{bmatrix} p_1(\mathbf{x}_1) & p_2(\mathbf{x}_1) & \ldots & p_m(\mathbf{x}_1) \\ p_1(\mathbf{x}_2) & p_2(\mathbf{x}_2) & \ldots & p_m(\mathbf{x}_2) \\ \ldots & \ldots & \ldots & \ldots \\ p_1(\mathbf{x}_n) & p_2(\mathbf{x}_n) & \ldots & p_m(\mathbf{x}_n) \end{bmatrix} \tag{7}$$

The quadratic basis functions $\mathbf{p}^T(\mathbf{x}) = [1\ x\ y\ x^2\ y^2\ xy]$ are used for numerical computations in this work. Vector $\mathbf{r}(\mathbf{x})$ in Eq. (3) is

$$\mathbf{r}(\mathbf{x}) = \{R(\mathbf{x}_1, \mathbf{x}) \quad R(\mathbf{x}_2, \mathbf{x}) \cdots R(\mathbf{x}_n, \mathbf{x})\}^T \tag{8}$$

where $R(\mathbf{x}_i, \mathbf{x})$ is the correlation function between the neighboring nodes $\mathbf{x}_i$ and $\mathbf{x}$, and it belongs to the covariance of field value $\mathbf{u}(\mathbf{x})$. The correlation matrix $\mathbf{R}$ with size $n \times n$ is given by

$$\mathbf{R}[R(\mathbf{x}_i, \mathbf{x}_j)] = \begin{bmatrix} 1 & R(\mathbf{x}_1, \mathbf{x}_2) & \ldots & R(\mathbf{x}_1, \mathbf{x}_n) \\ R(\mathbf{x}_2, \mathbf{x}_1) & 1 & \ldots & R(\mathbf{x}_2, \mathbf{x}_n) \\ \ldots & \ldots & \ldots & \ldots \\ R(\mathbf{x}_n, \mathbf{x}_1) & R(\mathbf{x}_n, \mathbf{x}_2) & \ldots & 1 \end{bmatrix} \tag{9}$$

Many different correlation functions can be used for $\mathbf{R}$. In this paper, a Gaussian function with a correlation parameter θ is used

$$R(\mathbf{x}_i, \mathbf{x}_j) = e^{-\theta r_{ij}^2} \tag{10}$$

in which $r_{ij} = ||\mathbf{x}_i - \mathbf{x}_j||$, and $\theta > 0$ is a correlation free parameter. As studied in the previous work [13–16], the correlation parameter has significant effect on the solutions. In this work, $\theta = 10.0$ is employed.

The partial derivatives of $\phi_I(\mathbf{x})$ against x_i can be obtained as following

$$\phi_{I,i}(\mathbf{x}) = \sum_{j=1}^{m} p_{j,i}(\mathbf{x})A_{jI} + \sum_{k=1}^{n} r_{k,i}(\mathbf{x})B_{kI} \tag{11}$$

where the index following a comma is a spatial derivative.

The Kronecker delta property of MK interpolation is demonstrated as following. The values of shape function $\phi_I(\mathbf{x})$ at points $\mathbf{x} = \mathbf{x}_J (J = 1, 2\cdots n)$ are

$$\phi_I(\mathbf{x}_J) = \sum_{j=1}^{m} p_j(\mathbf{x}_J)A_{jI} + \sum_{k=1}^{n} r_k(\mathbf{x}_J)B_{kI} \tag{12}$$

Eq. (12) can be written as following

$$\phi_I(\mathbf{x}_J) = \mathbf{PA} + \mathbf{RB} \tag{13}$$

Substituting Eq. (5b) into Eq. (13) gives

$$\phi_I(\mathbf{x}_J) = \mathbf{PA} + \mathbf{RR}^{-1}(\mathbf{I} - \mathbf{PA}) = \mathbf{I} \tag{14}$$

or

$$\phi_I(\mathbf{x}_J) = \delta_{IJ} \tag{15}$$

Then, the Kronecker delta property of MK interpolation is proved.

The accuracy of meshfree methods depends on the number of nodes for interpolation. In the previous works by authors [2–5], the number of nodes for interpolation is determined by the influence domain with a specified radius. And the number of interpolated nodes may be changed with different interest point $\mathbf{x}$. In this paper,