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Short communication

A viable method to predict acoustic streaming in presence of cavitation

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ABSTRACT

The steady liquid flow observed under ultrasonic emitters generating acoustic cavitation can be successfully predicted by a standard turbulent flow calculation. The flow is driven by the classical averaged volumetric force density calculated from the acoustic field, but the inertial term in Navier–Stokes equations must be kept, and a turbulent solution must be sought. The acoustic field must be computed with a realistic model, properly accounting for dissipation by the cavitation bubbles [Louisnard, Ultrason. Sonochem., 19, (2012) 56-65]. Comparison with 20 kHz experiments, involving the combination of acoustic streaming and a perpendicular forced flow in a duct, shows reasonably good agreement. Moreover, the persistence of the cavitation effects on the wall facing the emitter, in spite of the deflection of the streaming jet, is correctly reproduced by the model. It is also shown that predictions based either on linear acoustics with the correct turbulent solution, or with Louisnard's model with Eckart–Nyborg's theory yields unrealistic results.

1. Introduction

The propagation of acoustic waves (and of ultrasound in particular) in fluids is accompanied by steady flows, known as "acoustic streaming" [1–6]. The latter expression covers in fact various mechanisms, which can be divided into two main families, possibly occurring together: streaming near a solid boundary and streaming in unbounded fluid. The latter mechanism can be observed systematically in acoustic cavitation experiments, where a noticeable jet-like flow appears, as if it were expelled from the transducer.

The velocity fields of such flows have been evaluated either by visual observation [7], laser Doppler anemometry (LDA) [8,9], or particle image velocimetry (PIV) [10–13]. Using the latter method, Mettin and co-workers showed that the appearance of cavitation increased 30-fold the streaming velocities [14]. The corresponding flow was found to be turbulent, in agreement with the corresponding Reynolds number. Dubus and co-workers [15] mentioned that acoustic streaming currents generally hides the conical structure [16–18] visible under sonotrodes. They managed to suppress streaming by using pulsed ultrasound. Hihn and co-workers explored the combination of upward acoustic streaming above a transducer with a forced transverse horizontal flow in a rectangular duct [19,20]. They measured the deflection of the streaming jet by PIV as the velocity of the forced transverse flow was increased. Strangely enough, the authors found that even when the streaming jet was strongly deflected by the forced flow, cavitation remained active on the wall opposed to the transducer.

So far, no theory of acoustic streaming in presence of cavitation has been developed. In particular, there is no theoretical result available to quantitatively predict the velocities observed and explain why they are much larger in presence of cavitation.

The aim of this paper is to propose such a model, or rather an add-on to the model of wave propagation accounting for cavitation presented in Ref. [21]. The latter was found to predict correctly some yet incompletely explained bubble structures [18], and can be easily implemented in any geometry using COMSOL [22], for low frequency ultrasonics. It was shown in the latter references that our model, contrarily to linear acoustics and earlier cavitation models, correctly catches the strong wave attenuation near the emitter observable in presence of cavitation. As acoustic streaming is a matter of wave attenuation [6], it follows logically that a correct prediction of the latter is a pre-requisite for a viable evaluation of the former. Our model will be shown to compare reasonably well with the experiments reported in [20], and explains the persistence of cavitation even when the streaming jet is deflected.

2. Acoustic streaming models

The most popular model of acoustic streaming is attributed to Eckart [2], although Rayleigh [1], Westervelt [3] and Nyborg [4] contributed to the same result, and even found more general ones. As Eckart and Nyborg are names widely associated with streaming, we will refer to these results as Eckart–Nyborg's theory

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hereinafter. This model results from a regular perturbation procedure on compressible Navier–Stokes equations and exhibits the driving force for streaming as:

$$\mathbf{f} = -\nabla\left(\rho_{l_0}\overline{\mathbf{u}_1 \otimes \mathbf{u}_1}\right), \tag{1}$$

where $\mathbf{u}_1$ is the primary acoustic velocity field, ρ_{l_0} the liquid density at rest, overlined symbols denote averaged quantities over one acoustic period, and $\otimes$ is the dyadic product. The tensor in the divergence operator is the analog of the so-called "Reynolds stress tensor" in turbulence. In normal conditions (i.e. waves of moderate amplitude and no cavitation), $\mathbf{u}_1$ can be calculated from the equations of linear acoustics. Physically, the force density $\mathbf{f}$ represents an unbalance between the average (acoustic-induced) momentum entering and outgoing a given fluid volume. This unbalance must be compensated by a steady flow, which is precisely acoustic streaming.

The perturbation method followed by Eckart and Nyborg on mass and momentum conservation equations leads naturally to the following ones, governing the streaming velocity field:

$$0 = \nabla \cdot (\rho_{l_0}\mathbf{u}_{\mathrm{m}}) + \nabla \cdot (\overline{\rho_1 \mathbf{u}_1}), \tag{2}$$

$$0 = \mathbf{f} - \nabla p_{\mathrm{m}} + \mu_l \nabla^2 \mathbf{u}_{\mathrm{m}}, \tag{3}$$

where ρ_1 is the density variation associated with the primary acoustic field, all quantities with subscript m denote the steady flow defining acoustic streaming, μ_l is the liquid dynamic viscosity, and the force density $\mathbf{f}$ is defined by Eq. (1).

It can be noted that Eq. (3) is nothing else that the momentum equation of a creeping flow (driven by force $\mathbf{f}$), that is the reduction of Navier–Stokes equation to Reynolds numbers $\ll 1$. Lighthill, following this line of reasoning, argued that despite Eq. (1) is the correct expression of the driving force, its use in Eq. (3) reduces the applicability of Eckart–Nyborg's theory to very low acoustic intensities, for which the streaming velocities are low enough to fulfill $\mathrm{Re} \ll 1$. Cavitation experiments involve Reynolds numbers of several thousands [14], and lie therefore clearly outside this range of applicability. As an numerical illustration, consider for example a 10 mm sonotrode in water. A Reynolds number of 1 would correspond to a streaming characteristic velocity of 0.1 mm s^{-1}, which is by far much lower than commonly observed.

Lighthill suggested therefore that, rather than from Eq. (3), the streaming velocity $\mathbf{u}_{\mathrm{m}}$ should be calculated from the full steady Navier–Stokes equation (written here in conservative form):

$$\nabla\left(\rho_{l_0}\mathbf{u}_{\mathrm{m}} \otimes \mathbf{u}_{\mathrm{m}}\right) = \mathbf{f} - \nabla p_{\mathrm{m}} + \mu_l \nabla^2 \mathbf{u}_{\mathrm{m}}. \tag{4}$$

This equation was first derived by Zarembo [5] and defines what is generally termed as "Stuart streaming" [6,23], or "fast streaming" [24] as opposed to "slow streaming" which refers to Eckart–Nyborg's results.

The present model is based on the following hypothesis:

1. The first crucial assumption is to use magenta Stuart-Lighthill Eq. (4) rather than Eq. (3). Moreover, the acoustic streaming flows observed under cavitation are not only far from creeping flows, but are generally turbulent, as is clearly demonstrated in Refs. [8,14]. Thus, one should solve Eq. (4) for a turbulent flow. As is well-known, such flows presents small-scale eddies which are difficult, if not impossible, to solve directly by Navier–Stokes equations. Dedicated methods are therefore needed to seek turbulent solutions of Eq. (4).
2. On the other hand, the intensity of acoustic streaming is directly linked to the wave attenuation, as discussed in Ref. [6], [Section 4]. In this regard, it was shown in Ref. [25] that in presence of cavitation, the energy dissipated by the bubbles was the essential contribution to wave attenuation. This energy dissipation was calculated from numerically computed radial dynamics of inertial bubbles, allowing to simplify the model of Caflish et al. [26] into a nonlinear Helmholtz equation [21]. The resulting model was found to yield correct acoustic pressure levels in some typical configurations. More importantly, the strong wave attenuation was found to generate traveling waves [18], which are the only way to explain bubble strong ejection from the transducer [27]. The use of this realistic model of wave propagation is the second crucial point to the success of the present method.

Several theoretical predictions of acoustic streaming velocity fields in cavitating liquids have been proposed in the literature, either using Eckart–Nyborg Eq. (3) or Stuart-Lighthill Eq. (4). Most models pre-calculate the acoustic field in order to evaluate the driving force (1), generally by linear acoustics (possibly using a uniform attenuation coefficient as a free parameter) [28,29,10,11]. More complex but cavitation-unspecific acoustic models have also been tried [30]. Sajjadi and co-workers [31,32] derived simultaneously the acoustic and velocity fields from a time-dependent resolution of a two-phase model [33] describing the motion of a liquid containing vapor bubbles.[1] Kumar and co-workers [8,30] by-passed the computation of the acoustic field by using experimental LDA measurements of velocities and turbulence parameters as boundary conditions for the hydrodynamic problem. Trujillo & Knoerzer [23] proposed two original methods to compute the turbulent streaming jet without explicitly evaluating the acoustic field. The only input to their model is the input power, complemented by a single fitting parameter in each case. The methods follows closely Lighthill's derivation of the turbulent jet properties in the context of Stuart streaming, when either the driving force (1) can be considered as concentrated in one point, or a damped gaussian acoustic beam is considered. Surprisingly, these two simple and elegant methods show remarkable agreement with the experimental results of Kumar et al. [8].

The present model, even if it shares the use of Stuart-Lighthill Eq. (4) with some earlier models, differs from the latter in that our wave Eq. (5) accounts for the real energy disspation by an inertial bubble, rather than setting an empirical value to wave attenuation.

To conclude the discussion on acoustic streaming models, the derivation of Eq. (4) deserves a few comments. On the one hand, it requires a more subtle perturbation method than Eckart–Nyborg's theory, in order to avoid the natural disappearance of the inertial term in the left hand side [5,24]. On the other hand, there is no trivial justification of its validity within a cavitation model, which involves a two-phase flow. In other words, whether the driving force for streaming Eq. (1) is still valid with $\mathbf{u}_1$ calculated from our propagation model (which originates from Caflish model [26]) may be questioned. We will not enter more deeply in this discussion here, and it will be shown elsewhere that the set constituted by Caflish equations, Stuart-Lighthill Eq. (4 divergence-less field equation $\nabla \cdot \mathbf{u}_{\mathrm{m}} = 0$ instead of Eq. (2), can be recovered by a perturbation method performed on the Van Wijngaarden equations [35].

[1] Singhal's model [33] is a classical hydrodynamic cavitation model, normally restricted to bubbly liquid containing only vapor bubbles, which allows a simple closure of the two-phase equations. The assumptions made in this model normally prohibit its use for inertial acoustic cavitation bubbles, in spite of its increasing popularity in the latter context. Its use should be restricted to the special case where the transducer tip is covered entirely by the gas/vapor phase for long time intervals [34].

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