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An improved C_0 FE model for the analysis of laminated sandwich plate with soft core

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A B S T R A C T

An improved C_0 two dimensional finite element model based on higher order zigzag plate theory (HOZT) is developed and applied to the analysis of laminated composite and sandwich plates under different situations to study the performance of the model. In the proposed model, the in-plane displacements variation is considered to be cubic for both the face sheets and the core, while the transverse displacement is assumed to vary quadratically within the core and remains constant in the faces beyond the core. It satisfies the conditions of transverse shear stress continuity at the layer interfaces as well as satisfies the zero transverse shear stress condition at the top and bottom of the plate. The well-known problem of continuity requirement of the derivatives of transverse displacements is overcome by choosing the nodal field variables in an efficient manner. A nine-node C_0 quadratic plate finite element is implemented to model the HOZT for the present analysis. Numerical examples covering different features of laminated composite and sandwich plates are presented to illustrate the accuracy of the present model.

1. Introduction

A laminated composite/sandwich structure has a layered construction, which consists of a number of lamina or ply of orthotropic materials stacked one over the other and bonded together to act as an integral structural element. The individual layers of the laminate may have different orientations which enables the structural designer to achieve required strength in the preferred direction. It also shows superior properties such as high strength/stiffness to weight ratio and greater resistance to environmental degradation compared to conventional metallic materials. Due to all these merits, the fiber reinforced laminated composite/sandwich is gaining wide acceptance in various structural applications. In order to fulfill the requirement of weight minimization in a more efficient manner, a sandwich construction having low strength core and high strength face sheets is used.

The role of transverse shear deformation is very important in laminated composites, as the material is weak in shear due to its low shear modulus to extensional rigidity. Due to the large variation of material properties across the thickness, the behavior of laminated sandwich plates becomes very complex. A proper understating on the response of these structures under different loading conditions is extremely important for their safe design. In this context a number of plate theories have been developed where the major emphasis is to model the shear deformation in refined manner. These plate theories can be broadly divided into two categories based on their assumed displacement fields: (1) Single layer theory and (2) Layer-wise theory.

In single layer theory, the deformation of plate is expressed in terms of unknown parameters of the reference plane, i.e. middle plane. In this theory the transverse shear strain is assumed to be uniform over the entire plate thickness and it is known as Reissner–Mindlin's plate theory which is also known as the first order shear deformation theory (FSDT). Goyal and Kapania [1] developed a five node beam FE model based on FSDT. Moderately thick rectangular laminated composite plate was analyzed by Ferreira [2] using multiquadric radial basis function method (i.e. mesh free collocation method) based on FSDT. However, this theory (FSDT) requires a shear correction factor to compensate for the actual parabolic variation of the shear stress.

The higher order shear deformation theories (HSDT) have been developed with the aim to avoid the use of shear correction factors by including the actual cross sectional warping and to get the realistic variation of the transverse shear strains and stresses throughout the plate thickness [3]. Kant [4] derived the complete set of equations for the analysis of thick elastic plates with the help of third order refined shear deformation theory (HSDT). The three-dimensional Hooke's laws was also used for plate material

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in the theory [4] which gave a more realistic quadratic variation of the transverse shearing strains and linear variation of the transverse normal strain through the plate thickness. Later, Kant and Swaminathan [5] have reported analytical solutions based on the higher order shear deformation theory (HSDT). Using Reddy's displacement field [3] for third order shear deformation theory, a set of dynamic equations was derived for modeling of laminated structures by Aagaah et al. [6]. The theories proposed by Murthy et al. [7] and Subramanian [8] for the analysis of laminated beams, are also based on higher order shear deformation theory (HSDT). Pervez et al. [9] presented a two dimensional serendipity model based on HSDT for static analysis of laminated composite plate. Based on global local higher order shear deformation theory, Wu et al. [10] presented 4-node quadrilateral element and 3-node triangular element (assuming 13 field variables per node) satisfying weak continuity conditions for the static analysis of plates. Recently, Wu et al. [11] described the short coming of zigzag theories, i.e. requirement of C_1 continuity condition and presented a six node triangular C_0 FE by taking out the first derivatives of transverse displacements from the in-plane displacement fields for the static analysis of laminated sandwich plate. Kulkarni and Kapuria [12] proposed a new discrete Kirchhoff 4-node quadrilateral element (having 7 field variables per node) based on Reddy's HSDT [3]. The C_1 continuity requirement [12] is overcame by defining the derivatives of transverse displacement in terms of separate field variables. Aydogdu [13] presented HSDT for the static, vibration and buckling analysis of laminated composite plates where the shear deformation function was chosen according to 3-D analysis by using inverse method. A nine-node rectangular element with nine field variables at each node was developed by Tu et al. [14] based on HSDT for the bending and vibration analysis of laminated composite and sandwich plates. Ferreira et al. [15] presented the radial basis function collocation method for the static and vibration analysis of thick plates using FSDT and HSDT given by Kant [4]. For the analysis of thin and thick composite plates, Roque et al. [16] used higher order shear deformation theory. Due to different values of shear rigidity at the adjacent layers, HSDT shows discontinuity in the shear stress distribution at the layer interfaces with continuous variation of the transverse shear strain across the thickness. But the actual behavior of a composite laminate is opposite, i.e., the transverse shear stress must be continuous at the layer interface and the corresponding strain may be discontinuous [17].

In order to overcome the above disparity, the layer-wise theories are developed. The layer-wise theories may be further classified into discrete layer plate theory and refined plate theory. In discrete layer plate theory, unknown displacement components are taken at all the layer interfaces. Discrete layer theories proposed by Robbins and Reddy [18], Toledano and Murakami [19], Lu and Liu [20], Reddy [21] and many others assume unique displacement field in each layer and displacement continuity across the layers. Tahani [22] presented the analytical solution for laminated beams by using two theories based on layer-wise displacement fields. Ramesh et al. [23] presented a 45-node triangular element with 7 and 3 field variables at each node, based on the HSDT and layer-wise plate theory of Reddy respectively for the static analysis of the laminated composite plate. The performance of this plate theory is good but it required huge computational involvement as the number of unknowns increases directly with the increase in the number of layers.

To solve the above problem, the unknowns at different interfaces are defined in terms of those at the reference plane in refined plate theories (also known as zigzag theories). In this theory, the in plane displacements have piecewise variation across the plate thickness and the number of unknowns are made independent of the number of layers by equating the transverse shear stresses at the layer interfaces of the laminate. In some improved version of these theories, the condition of zero transverse shear stresses at the plate/beam top and bottom was also satisfied. The theories developed by Murakami [24], Di Sciuva [25], Lee et al. [26], Cho and Parmerter [27], Cho and Averill [28] and many other fall under this category. Carrera [29] presented a historical review on the zigzag theories used for the analysis of multilayered laminate plates and shells, in which the three basic theories have been discussed, namely: Lekhnitskii Multilayered Theory (LMT), Ambartsumian Multilayered Theory (AMT) and Reissner Multilayered Theory (RMT). A triangular element was presented by Chakrabarti and Sheikh [30] based on zigzag theory, which shows excellent performance though the element is does not satisfy the normal slope continuity requirement. Akhras and Li [31] developed a spline finite strip method based on higher order zigzag theory for the static analysis of the plate. Kapuria and Kulkarni [32] presented a four node quadrilateral element based on third order zigzag theory for the analysis of the laminates. The C_1 continuity requirement is circumvented by using discrete Kirchoff constraint approach, where the derivaties of transverse displacement are replaced by rotational variables. Fares and Elmarghany [33] presented a first order zigzag theory of composite plates using Ressiner's mixed variational formula. Recently, Ferreira et al. [34] and Rodrigues et al. [35] presented radial basis functions, finite differences collocation and unified formulation for the analysis of laminated plates based on zigzag theory. Zigzag models for laminated composite structures were developed by using trigonometric terms to represent the linear displacement field, transverse shear strains and stresses [36–38]. These theories (Zigzag) provide a very accurate approximation of the structural behavior even for lower span to thickness ratio. However, the zigzag theory has a problem in its finite element implementation as it requires C_1 continuity of the transverse displacement at the nodes.

To combine the benefits of the discrete layer-wise and higher order zigzag theories, Icardi [39,40], Yip and Averill [41] and many others developed theories which are known as sub-laminated models. Cho and Averill [42] presented an improved sub-laminate model with first order zigzag approximation of displacement within each sub-laminate, which contains an eight node C_0 FE having five displacement field variables at each node for each sub-laminate. Averill [43] developed a C_0 finite element based on first order zigzag theory and overcome the C_1 continuity requirement by incorporating the concepts of independent interpolations and penalty functions. Hermitian functions were used by Di Sciuva [44,45] to approximate the transverse displacement in his formulations. Carrera [46] used two different fields along the laminate thickness direction for transverse displacement and transverse shear stress respectively for formulation. Averill and Yip [47] developed a C_0 finite element based on cubic zigzag theory, using interdependent interpolations for transverse displacement and rotations and penalty function concepts. Aitharaju and Averill [48] developed a new C_0 FE based on a quadratic zigzag layer-wise theory. For eliminating shear locking phenomenon, the shear strain field is also made field consistent. The transverse normal stress was assumed to be constant through the thickness of the laminate. The new FE was applied to model the beam as combination of different sub-laminates.

A C_0 plate model based on enhanced first order theory (EFSDT) was presented by Kim and Cho [49,50], where it was shown that the displacements, in-plain strains and stresses can be approximated to those of the three dimensional theory or higher order theory, in the least square sense. Recently the authors [51], have developed a C_0 model using the EFSDT based on mixed variational theorem, which also satisfy the lateral conditions at the top and bottom surfaces of the plate. The mixed FE approach was