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On the vibration and stability of spinning axially loaded pre-twisted Timoshenko beams

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ABSTRACT

Two sets of lateral vibration equations for a spinning axially loaded twisted Timoshenko beam have been studied. The compressed axial load is assumed to be normal to the shear force and tangential to the axis of the beam for the two systems, respectively. A quadratic eigenvalue problem of a real gyroscopic system is formulated and utilized to investigate the free vibration and buckling stability of various twisted Timoshenko beams. Some typical results are compared with numerical results in the published literature to validate the accuracy of the presented analysis. The influence of thickness-to-width ratio, twist angle, spinning speed and axial load on the natural frequency and buckling load of Timoshenko beams has been investigated and discussed. Comparisons between the results of the two sets of system equations are also made to justify the effect of the axial load for various Timoshenko beams.

1. Introduction

Vibration and stability analyses of non-rotating and rotating beams with or without axial loads have been investigated extensively since the dynamic problem of beams is important in the design of machines and structures. Typical examples include rotating shafts, satellite booms, turbine blades, drill bits, end-milling cutters and boring bars. Spinning straight circular beams are often used to model the rotating shafts and satellite booms. Pre-twisted beams and spinning pre-twisted beams have been utilized to analyze successfully the dynamic behavior of turbine blades and fluted cutters, respectively. During drilling or milling process, the fluted cutters are always subjected to axial forces. Thus, the effect of axial load has to be considered in the vibrational analysis of fluted cutters in addition to the rotational speed. Generally, the problem of vibration behavior of the untwisted and twisted beam structures has been analyzed based on the Euler beam theory or the Timoshenko beam theory. A general review of the dynamic aspects of twisted beams can be found in review paper by Leissa [1] and Rosen [2].

Based on the Euler–Bernoulli beam theory, the effect of the twist angle on the natural frequencies and mode shapes of the cantilever turbine blade had been studied in [3–5]. Rayleigh's method was used by Carnegie [3] to evaluate the bending frequencies and mode shapes of pre-twisted cantilever rectangular beams. The results revealed that the pretwist slightly increases the fundamental frequency. The numerical integration method of solving the system equations of motion of first-order was applied by Dawson and Carnegie [4] to find the modal curves of pre-twisted rectangular beams with different width to depth ratios and pretwist angles. The theoretical results were in good agreement with the experimental ones. A finite element model with a cubic polynomial displacement function was presented by Sabuncu [5] to analyze the bending vibration of pre-twisted blading with uniform cross section. In comparison with results between linearly and nonlinearly pre-twisted beams, the deviations increased as increasing pretwist angle. By treating the blade as a pre-twisted Timoshenko beam, the vibration equations of motion of the pre-twisted blade were developed using different techniques [6–13] to study the effects of geometric aspects, rotary inertia and shear deformation on the lateral frequencies of the blade. The variational method was utilized by Carnegie [6] to derive the bending–bending–torsion equations of motion for a pre-twisted cantilever blade allowing for the shear deformation and rotary inertia effects. The transformation method was used by Dawson et al. [7] to investigate the effect of slenderness ratio on the natural frequencies of pre-twisted cantilever Timoshenko beams. Theoretical results were higher than the experimental data and the discrepancy increased as the beam length decreased. The finite element method was employed by Gupta and Rao [8] to evaluate the natural frequencies of uniformly pre-twisted tapered cantilever Timoshenko beams. The effects of shear deformation and rotary inertia reduce the frequencies, especially for higher modes of vibration. A simple finite element model was presented by Abbas [9] for dynamic analysis of thick pre-twisted blades. The simple model gave close approximation of natural frequencies. The Reissner method and the total potential energy approach

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Nomenclature

A	cross-sectional area
b	width of the beam
$\mathbf{C}$	global Coriolis matrix
$\mathbf{C}^{(e)}$	element Coriolis matrix
$\overline{\mathbf{C}}$	Coriolis coefficient matrix
$\mathbf{d}$	displacement matrix in frame $\xi\eta z$
$\mathbf{d}^{(e)}$	displacement function of the beam
E	Young's modulus
F_z	axial load
$\mathbf{F}=-\mathbf{KF}/F_z$	global stiffness matrix due to unit axial force
$F^{(e)}=-K_F^{(e)}/F_z$	element stiffness matrix due to unit axial force
G	shear modulus
h	thickness of the beam
I_{XX}, I_{YY}, I_{XY}	area moments and product of inertia in frame XYZ
I_ξ, I_η	principal area moments of inertia in frame $\xi\eta z$
J_{XX}, J_{YY}, J_{XY}	mass moments and product of inertia per unit length in frame XYZ
J_ξ, J_η	principal mass moments of inertia per unit length in frame $\xi\eta z$
$\mathbf{K}$	global stiffness matrix
$\overline{\mathbf{K}}_1,\overline{\mathbf{K}}_2,\overline{\mathbf{K}}_3,\overline{\mathbf{K}}_4$	stiffness coefficient matrices due to the bending and shear effects
$\mathbf{K}_B$	global stiffness matrix due to the bending and shear effects
$K_B^{(e)}$	element stiffness matrix due to the bending and shear effects
$\mathbf{K}_F$	global stiffness matrix due to axial force
$K_F^{(e)}$	element stiffness matrix due to axial force
$\mathbf{K}_\Omega$	global stiffness matrix due to spinning speed
$K_\Omega^{(e)}$	element stiffness matrix due to spinning speed
$\overline{K}_\Omega$	stiffness coefficient matrix due to spinning speed
L	beam length
Le	beam element length
m	beam mass per unit length
$\mathbf{M}$	global inertia matrix
$\mathbf{M}^{(e)}$	element inertia matrix
$\overline{\mathbf{M}}$	inertia coefficient matrix
$\mathbf{N}$	transformation matrix between displacement function and nodal displacements
$N1$, $N2$	shape functions of linear beam element
P_{cr}	buckling load
p	F_zL^2/EI, axial load parameter
p_{cr}	$P_{cr}L^2/EI_\xi$, buckling load parameter
$\mathbf{p}$	global displacement matrix
$\mathbf{p}^{(e)}$	element displacement matrix
$\mathbf{q}$	constant vector
r^2	I/AL^2, rotary inertia parameter
s^2	$Er^2/G\kappa$, shear deformation parameter
T	kinetic energy
u_X, u_Y	total transverse displacements in XYZ frame
u_ξ, u_η	transverse displacements in $\xi\eta z$ frame
$u_{\xi1}$, $u_{\eta1}$, $u_{\xi2}$, $u_{\eta2}$	nodal transverse displacements in $\xi\eta z$ frame
V	potential energy
W	work produced by the axial load
z	axial coordinate
κ	shear correction factor
ρ	density of the beam
λ	$(\rho A\omega^2L^4/EI_\xi)^{1/4}$, bending frequency parameter
ω	natural frequency
$\overline{\omega}$	$\omega(\rho AL^4/(EI_\xi EI_\eta)^{1/2})^{1/2}$, dimensionless natural frequency
ω_{EU}	first bending frequency of the untwisted Euler–Bernoulli beam
Ω	spinning speed of the beam
$\overline{\Omega}$	$\Omega(\rho AL^4/(EI_\xi EI_\eta)^{1/2})^{1/2}$, dimensionless spinning speed
Ω^*	Ω/ω_{EU}, normalized spinning speed
β_o	twist angle per unit length
ϕ	total twist angle
φ_x, φ_y	angles of rotation in XYZ frame
φ_ξ, φ_η	angles of rotation in $\xi\eta z$ frame
$\varphi_{\xi1}$, $\varphi_{\eta1}$, $\varphi_{\xi2}$, $\varphi_{\eta2}$	nodal angles of rotation in $\xi\eta z$ frame

were used by Subrahmanyam et al. [10] to obtain the natural frequencies of pre-twisted cantilever blades. The results indicated that the Reissner method gives a faster convergence than the potential energy method. Based on Hamilton's principle, the bending–bending forced vibration equations of motion were established by Lin et al. [11] for a nonuniformly pre-twisted Timoshenko beam with general elastic boundary conditions. The taper ratio and spring constants have a greater influence on the higher-mode natural frequencies; the nonuniform pretwist has a greater effect on the natural frequencies than uniform pretwist. A dynamic stiffness matrix was developed and used by Banerjee [12] to study the free vibration of a twisted Timoshenko beam. The effects of shear deformation and rotary inertia on the natural frequencies of a twisted beam are same as those of the corresponding straight beam. Based on coupled displacement fields, a new finite element model was developed and used by Yardimoglu and Yildirim [13] to determine the natural frequencies of pre-twisted rectangular Timoshenko beams. The authors indicated that the new pre-twisted Timoshenko beam element possesses good convergence characteristics. Above studies were concentrated mainly on the free vibration characteristics of non-rotating pre-twisted beam structures.

The dynamic behavior of rotating beams about a longitudinal or transverse axis had been investigated extensively related to the vibration of shafts, turbine blades and drill bits. The influence of the rotational speed on natural frequencies and stability of untwisted beams spinning about the longitudinal axis has been studied in [14–20]. An analytical method was presented by Bauer [14] to investigate the free vibration of spinning Euler straight beams with various end conditions. The natural frequencies either decrease or increase linearly as the spinning speed is increased. Dynamic stability of spinning Euler beams of unsymmetrical cross-section with distinct end conditions was dealt with by Lee [15]. The spinning beams are found to have distinct stable and unstable spinning speed regimes separated by critical spinning speeds. The natural bending frequency of a spinning cylindrical shaft was investigated by Behzad and Bastami [16]. Numerical results showed that the axial force produced by shaft rotation significantly affects the natural frequency of long shafts at high spinning speed. The dynamic stiffness method was used by Banerjee and Su [17] to analyze the free vibration of spinning Euler beams with doubly symmetric cross section. The natural frequencies of spinning beams with circular and rectangular cross-sections were obtained by using the Wittrick–Williams algorithm. The frequency equations and critical speeds of spinning straight circular Timoshenko beams were obtained by Eshleman and Eubanks [18]. The shear deformation effect, rotary inertia and gyroscopic moment were considered. Finite element methods were presented by Nelson [19], and Gmur and Rodrigues