Finite Elements in Analysis and Design 47 (2011) 496–510

Contents lists available at ScienceDirect

Finite Elements in Analysis and Design

journal homepage: www.elsevier.com/locate/finel

Interactive visualization of elasto-plastic behavior through stress paths and yield surfaces in finite element analysis

Jae Young Lee [a,*], Sung Youll Ahn [b]

[a] Department of Regional Construction Engineering, Chonbuk National University, Jeonju 561-756, Republic of Korea
[b] SytecEnc Co., Ltd., 796 Jayang-dong Gwanjin-gu, Seoul 143-190, Republic of Korea

A R T I C L E I N F O

Article history:
Received 10 March 2010
Received in revised form
18 November 2010
Accepted 10 December 2010
Available online 3 February 2011

Keywords:
Finite element analysis
Post-processing
Elasto-plastic analysis
Yield criterion
Yield surface
Stress path

A B S T R A C T

This paper proposes a new approach for representing elasto-plastic behavior through graphical visualization of the yield surface and the stress path in the principal stress space.

The yield surface is defined by the material parameters of a given yield criterion, and the stress path is formed by a number of stress points, each of which is obtained for every increment in the applied force. The superposed images of the yield surface and the stress path can be used as a tool for interpreting the results of finite element analysis based on the assumed elasto-plastic yield theory.

The paper describes how to construct the stress path and the yield surface by following procedures ranging from data extraction to graphical rendering. It suggests a framework for the interactive control of visualization and data sampling. Geometric modeling of the yield surface is formulated for various failure criteria that are frequently used in finite element analysis.

Examples of stress path and yield surface visualization are presented herein to demonstrate the practical use of this approach.

All of the features described in this paper can be implemented easily as a post-processing capability in finite element analysis programs.

1. Introduction

The ever more complex and delicate modeling of structural problems by the finite element method owes to ever-increasing computing power. A consequent issue is how to manage and utilize the huge amount of numerical data produced by the result [1]. In this regard, finite element post-processing has assumed the important role of making the data more accessible and understandable and thus facilitating the interpretation and application of the results of the analysis. This objective can be achieved by transforming the numerical data into graphical images representing physical phenomena that can be identified by well-known theoretical concepts [2–4]. Various techniques of graphical rendering have been developed to express the different forms of data such as scalar, vector, flow, etc. [5].

The time required for an iteration of equation solving has been reduced remarkably by the improvement of computational algorithms as well as the acceleration of hardware speeds. In turn, this allows a significant increase in the number of processing steps in nonlinear finite element analysis in the interest of superior accuracy. At the same time, the extended capacity of computer storage enables the recording and retrieval not only of the results at the final step but also of those created at intermediate steps [6].

This paper is concerned with the post-processing of data that are obtained in stepwise processes of nonlinear finite element analysis. It proposes a new method for visualizing the results of multi-step analysis in view of the yield surface and stress path so that the elasto-plastic behavior can be figured quantitatively from the solved model.

The yield surface represents the yielding characteristics that constrain stress development at a specific point of the model for analysis, and it is determined by the yield criterion and other related material constants that are assumed at that point. The stress path is the record of stresses that are developed at the same point due to incrementally applied forces. Thus, the elasto-plastic behavior of the model can be construed from the results of analysis by viewing the stress path against the constraining yield surface, both of which are plotted in the principal stress space.

The stress path is the collection of stress points. A stress point represents the principal stress components as a coordinate in the principal stress space. As the applied forces increase, the stresses change. In accordance, the stress point moves away from the origin of the principal coordinates, and forms a stress path. The elasto-plastic behavior of an object that is subjected to an external force can be visualized by the image of the stress path that is superposed on that of the yield surface.

* Corresponding author. Tel.: +82 63 270 2520, Mobile: +82 10 8645 8919; fax: +82 63 270 2517.
E-mail address: jylee@jbnu.ac.kr (J.Y. Lee).

0168-874X/$ - see front matter © 2010 Elsevier B.V. All rights reserved.
doi:10.1016/j.finel.2010.12.017

Lee [7] suggested the various possibilities of graphically visualizing elasto-plastic behavior simulated by finite element analysis. Based on such a concept of finite element post-processing, this study presents graphical modeling and rendering of the yield surface and stress path as a means of representing elasto-plastic behavior. This paper describes the following topics related to the realization of the suggested approach:

- Extraction of data for visualization.
- Transformation of the data into stress coordinates.
- Construction of the yield surface or the envelope curve.
- Construction of the stress path.
- Rendering of the yield surface (or the envelope curve) and the stress path.
- Interactive user interface.

Examples are also presented at the end of the paper demonstrating practicality and providing guidance as to the actual application of the work presented herein.

The methods suggested in this paper can be implemented in a finite element analysis program [8,9], with emphasis on the ease of use and interactiveness.

The validity and practicality of these methods have been established through application and testing in actual finite element processing. The graphic visualization of elasto-plastic behavior can also be employed as an educational aid in the related topics of structural or soil mechanics [10].

2. Data extraction for visualization of the stress path and yield surface

The stress path and yield surface are constructed based on data that are obtained from actual finite element analysis in lieu of stress data that are arbitrarily assumed. All of the data necessary for visualization are prepared automatically by going through a cycle of finite element analysis. Alternatively, this procedure for analysis may be replaced by reading the data from a previously solved modeling file. Such an approach may be practical for most finite element analysis systems integrated with modeling and analysis capabilities including pre- and post-processing.

The data flow from modeling through to visualization is summarized in Fig. 1.

2.1. Data necessary for rendering the stress path and yield surface

The stress path is formed by plotting a series of stress points that represent the principal stresses at different levels of force application. The principal stresses are derived from the normal and shear stress components.

The yield surface is determined by the yield criterion and related material parameters, as listed in Table 1, and these parameters are specified at the sampling point and stored in the modeling database. Additional data may be required depending on the yield criterion. Rendering of the Cam-clay yield surface, for example, involves the pre-consolidation pressures that are usually specified as *in-situ* stresses. The material parameters necessary for rendering of the yield surfaces classified in Section 3.2 are listed in Table 1.

The material properties and stresses vary from one point to another. It is necessary to set a consistent scale for the rendering of the yield surface so that the display area of the screen can accommodate the image for any sampling point within the model.

The minimum and maximum range of the stress magnitude should be known in order to set the scale and extent of the rendering of the yield surface appropriately.

2.2. Data extraction at the sampling point

It is assumed that the stress distribution is represented by the nodal values, which are obtained through the stress recovery and smoothing processes. The value of the data can then be approximated at the designated sampling point by relating its position to the surrounding nodal coordinates. The sampling point is chosen simply by a mouse click over the desired position within the model for analysis. We may think of the following three types of selected objects:

- *Planar or surface element*:
  The position of the sampling point is identified by the encompassing element and its natural coordinates which are estimated simply from the screen coordinates of the clicked point and the nodal points of the element.
  In the case of an n-noded planar element, for example, the natural coordinates (ξ,η) of the sampling point can be obtained

Table 1
Material parameters for yield surface rendering.

Geomtetric characteristic	Yield criterion	Material parameters
Uniform profile	Von mises Tresca	Yield strength
Straight-lined profile	Mohr–Coulomb Drucker–Prager	Angle of internal friction, cohesion
Curved profile	Hoek–Brown	Uniaxial compressive strength, geological strength index, disturbance factor, intact rock parameter
Closed profile	Cam-clay Modified Cam-clay	Critical state void ratio, critical state friction constant

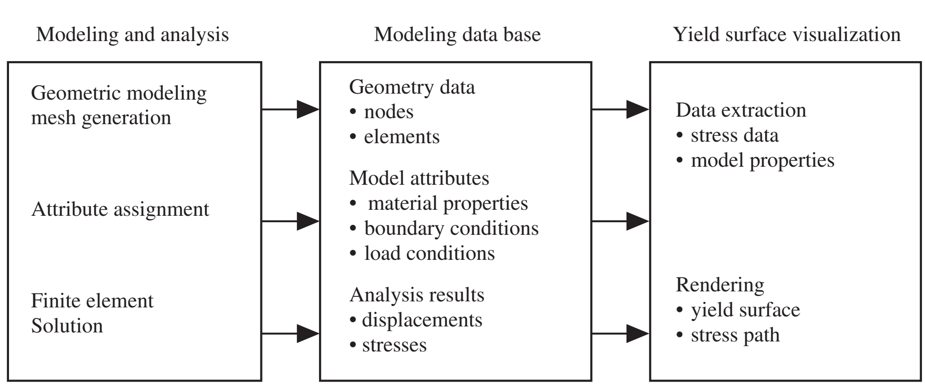

Fig. 1. Data flow for the visualization of the stress path and yield surface.