Finite Elements in Analysis and Design 47 (2011) 519–535

Contents lists available at ScienceDirect

Finite Elements in Analysis and Design

journal homepage: www.elsevier.com/locate/finel

An alternative alpha finite element method with discrete shear gap technique for analysis of isotropic Mindlin–Reissner plates

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ARTICLE INFO

Article history:
Received 24 March 2010
Received in revised form
10 December 2010
Accepted 1 January 2011
Available online 5 February 2011

Keywords:
Finite element method
Smooth finite element method (SFEM)
Discrete shear gap (DSG)
Transverse shear locking
Plate bending

ABSTRACT

An alternative alpha finite element method (AαFEM) coupled with a discrete shear gap technique for triangular elements is presented to significantly improve the accuracy of the standard triangular finite elements for static, free vibration and buckling analyses of Mindlin–Reissner plates. In the AαFEM, the piecewise constant strain field of linear triangular elements is enhanced by additional strain terms with an adjustable parameter α which results in an effectively softer stiffness formulation compared to the linear triangular element. To avoid the transverse shear locking, the discrete shear gap technique (DSG) is utilized and a novel triangular element, the A$\alpha-$DSG3 is obtained. Several numerical examples show that the A$\alpha-$DSG3 achieves high reliability compared to other existing elements in the literature. Through selection of α, under or over estimation of the strain energy can be achieved.

1. Introduction

The finite element analysis of plate structures plays an important role in engineering applications because the plate is one of the most widely used structural components. In practical applications, lower order Mindlin–Reissner plate elements are preferred due to their simplicity and efficiency. However, these low-order plate elements in the limit of thin plates often suffer from the shear locking phenomenon which has the root of incorrect transverse forces under bending.

Therefore, many formulations have been developed to overcome the shear locking phenomenon and to increase the accuracy and stability of numerical methods such as mixed formulation/hybrid elements [1–4], the enhanced assumed strain (EAS) method [5,6] and the assumed natural strain (ANS) method [7,8]. Recently, the discrete shear gap (DSG) method [9] which can avoid shear locking was proposed. The DSG is similar to the ANS methods in the aspect of modifying the course of certain strains within the element, but different in that it does not employ collocation points, which makes the DSG method independent of the order and shape of the element.

The smoothed FEM (SFEM) [10] based on strain smoothing is ideally suited to extremely distorted meshes. Another advantage of the SFEM is that derivatives of the shape functions are not required, leading to lower computational cost because of the absence of an isoparametric mapping. The SFEM has also been extended to general n-sided polygonal elements (nSFEM) [11], dynamic analysis [12–14], plate and shell analysis [15–19] and coupled to partition of unity enrichment [20–25]. The latter paper also provides a review of strain smoothing in FEM. A general framework for this strain smoothing technique in FEM was proposed in [26]. Based on the idea of the node-based smoothed point interpolation method (NS-PIM) and the SFEM, a node-based smoothed finite element method (NS-FEM)[27] for 2D solid mechanics problems has been developed.

Recently, Liu et al. [28,29] have proposed a superconvergent alpha finite element method (S α FEM) using triangular meshes. Nguyen-Thanh et al. [30] extended the AαFEM to free and forced vibration analyses of solid 2D mechanics problems. In the AαFEM, an assumed strain field was formulated by adding the averaged nodal strains with an adjustable factor α to the compatible strains. The new Galerkin-like weak form, as simple as the Galerkin weak form, was then obtained for this constructed strain field. It was proven theoretically and numerically that the AαFEM is always more accurate than the original FEM-T3 as well as the FEM-Q4 when the same sets of nodes are used.

In this paper, we further extend the AαFEM to static, free vibration and buckling analyses of Mindlin–Reissner plates using triangular elements only. In the AαFEM for plates, the bending, shearing and geometrical stiffness matrices of the standard FEM

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doi:10.1016/j.finel.2011.01.004

formulation are enhanced by additional strain terms with an adjustable parameter α which results in an effectively softer stiffness formulation compared to the linear triangular element. Transverse shear locking can be avoided through the discrete shear gap (DSG) method. Several numerical examples illustrate the high performance of the Aα–DSG3 formulation compared to other elements from the literature.

The paper is arranged as follows. The next section describes the discrete governing equations. In Section 3, an assumed strain field based on linear triangular elements (T3) is introduced. Next, some theoretical properties of the AαFEM is presented. Section 5 presents and discusses numerical results. We close our paper with some concluding remarks and ideas for future work.

2. Discrete governing equations

Let Ω be the domain in $\mathbb{R}^2$ occupied by the mid-plane of the plate and w and $\boldsymbol{\beta}=(\beta_x,\beta_y)^T$ denote the transverse displacement and the rotations in the x–z and y–z planes, see Fig. 1, respectively. Assuming that the material is homogeneous and isotropic with Young's modulus E and Poisson's ratio ν, the governing differential equations of the Mindlin–Reissner plate are given by

$$-\mathrm{div}\mathbf{D}^b\boldsymbol{\varepsilon}^b(\boldsymbol{\beta})-\lambda t\boldsymbol{\varepsilon}^s(\boldsymbol{\beta})=0 \quad \text{in } \Omega \tag{1}$$

$$-\lambda t\,\mathrm{div}(\boldsymbol{\varepsilon}^s)=p \quad \text{in } \Omega \tag{2}$$

$$w=\overline{w},\boldsymbol{\beta}=\overline{\boldsymbol{\beta}} \quad \text{on } \Gamma=\partial\Omega \tag{3}$$

where t is the plate thickness, $p=p(x,y)$ is the transverse loading per unit area, $\lambda=kE/2(1+\nu)$, $k=5/6$ is the shear correction factor and $\mathbf{D}^b$ (Eq. (11)) is the tensor of bending moduli. The bending $\boldsymbol{\varepsilon}^b$ and shear strains $\boldsymbol{\varepsilon}^s$ are defined as

$$\boldsymbol{\varepsilon}^b=\mathbf{L}_d\boldsymbol{\beta}, \quad \boldsymbol{\varepsilon}^s=\nabla w+\boldsymbol{\beta} \tag{4}$$

where $\nabla=(\partial/\partial x,\partial/\partial y)$ is the gradient vector and $\mathbf{L}_d$ is a differential operator matrix defined by

$$\mathbf{L}_d^T=\begin{bmatrix}\partial/\partial x & 0 & \partial/\partial y\\ 0 & \partial/\partial y & \partial/\partial x\end{bmatrix} \tag{5}$$

The weak form of the static equilibrium equations in Eq. (3) is as follows:

$$\int_\Omega(\delta\boldsymbol{\varepsilon}^b)^T\mathbf{D}^b\boldsymbol{\varepsilon}^b\,d\Omega+\int_\Omega(\delta\boldsymbol{\varepsilon}^s)^T\mathbf{D}^s\boldsymbol{\varepsilon}^s\,d\Omega=\int_\Omega\delta\mathbf{u}^T\overline{\mathbf{p}}\,d\Omega \tag{6}$$

where the displacement field is given by $\mathbf{u}=[w,\beta_x,\beta_x]^T$, and the transverse load is redefined by $\overline{\mathbf{p}}=[p,0,0]^T$.

For the free vibration analysis of a Mindlin–Reissner plate model, a weak form may be derived from the dynamic form of the principle of virtual work under the assumptions of first order shear-deformation plate theory.

$$\int_\Omega(\delta\boldsymbol{\varepsilon}^b)^T\mathbf{D}^b\boldsymbol{\varepsilon}^b\,d\Omega+\int_\Omega(\delta\boldsymbol{\varepsilon}^s)^T\mathbf{D}^s\boldsymbol{\varepsilon}^s\,d\Omega+\int_\Omega\delta\mathbf{u}^T\mathbf{m}\ddot{\mathbf{u}}\,d\Omega=0 \tag{7}$$

In the case of in-plane buckling analyses and assuming pre-buckling stresses $\hat{\boldsymbol{\sigma}}_0$, non-linear strains appear and the weak form can be reformulated as [31]

$$\int_\Omega(\delta\boldsymbol{\varepsilon}^b)^T\mathbf{D}^b\boldsymbol{\varepsilon}^b\,d\Omega+\int_\Omega(\delta\boldsymbol{\varepsilon}^s)^T\mathbf{D}^s\boldsymbol{\varepsilon}^s\,d\Omega+t\int_\Omega\nabla^T\delta w\hat{\boldsymbol{\sigma}}_0\nabla w\,d\Omega$$
$$+\frac{t^3}{12}\int_\Omega[\nabla^T\delta\beta_x\quad\nabla^T\delta\beta_y]\begin{bmatrix}\hat{\boldsymbol{\sigma}}_0 & 0\\ 0 & \hat{\boldsymbol{\sigma}}_0\end{bmatrix}\begin{bmatrix}\nabla\beta_x\\ \nabla\beta_y\end{bmatrix}d\Omega=0 \tag{8}$$

Eq. (8) can be rewritten as

$$\int_\Omega(\delta\boldsymbol{\varepsilon}^b)^T\mathbf{D}^b\boldsymbol{\varepsilon}^b\,d\Omega+\int_\Omega(\delta\boldsymbol{\varepsilon}^s)^T\mathbf{D}^s\boldsymbol{\varepsilon}^s\,d\Omega+\int_\Omega(\delta\boldsymbol{\varepsilon}^g)^T\boldsymbol{\tau}\boldsymbol{\varepsilon}^g\,d\Omega=0 \tag{9}$$

where

$$\boldsymbol{\varepsilon}=\begin{bmatrix}\boldsymbol{\varepsilon}^b\\ \boldsymbol{\varepsilon}^s\end{bmatrix}=\begin{bmatrix}\beta_{x,x}\\ \beta_{y,y}\\ \beta_{x,y}+\beta_{y,x}\\ \beta_x+w_{,x}\\ \beta_y+w_{,y}\end{bmatrix},\quad \boldsymbol{\varepsilon}^g=\begin{bmatrix}w_x & 0 & 0\\ w_y & 0 & 0\\ 0 & \beta_{x,x} & 0\\ 0 & \beta_{x,y} & 0\\ 0 & 0 & \beta_{y,x}\\ 0 & 0 & \beta_{y,y}\end{bmatrix},$$
$$\boldsymbol{\tau}=\begin{bmatrix}t\hat{\boldsymbol{\sigma}}_0 & 0 & 0\\ 0 & \frac{t^3}{12}\hat{\boldsymbol{\sigma}}_0 & 0\\ 0 & 0 & \frac{t^3}{12}\hat{\boldsymbol{\sigma}}_0\end{bmatrix} \tag{10}$$

$$\hat{\boldsymbol{\sigma}}_0=\begin{bmatrix}\boldsymbol{\sigma}_x^0 & \boldsymbol{\sigma}_{xy}^0\\ \boldsymbol{\sigma}_{xy}^0 & \boldsymbol{\sigma}_y^0\end{bmatrix},\quad \mathbf{D}^b=\frac{Et^3}{12(1-\nu^2)}\begin{bmatrix}1 & \nu & 0\\ \nu & 1 & 0\\ 0 & 0 & \frac{1-\nu}{2}\end{bmatrix},$$
$$\mathbf{D}^s=k\frac{Et}{2(1+\upsilon)}\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} \tag{11}$$

Let us assume that the bounded domain Ω is discretized into nel finite elements such that $\Omega=\bigcup_{e=1}^{nel}\Omega^e$ and $\Omega^i\neq\Omega^j\neq\emptyset$, $i\neq j$. The finite element solution $\mathbf{u}^h$ of a displacement model for the Mindlin–Reissner plate is then expressed as

$$\mathbf{u}^h=\sum_{I=1}^{np}\begin{bmatrix}N_I(\mathbf{x}) & 0 & 0\\ 0 & N_I(\mathbf{x}) & 0\\ 0 & 0 & N_I(\mathbf{x})\end{bmatrix}\mathbf{d}_I \tag{12}$$

where np is the total number of nodes $N_I(\mathbf{x})$, $\mathbf{d}_I=[w_I\ \theta_{xI}\ \theta_{yI}]^T$ are shape functions and the nodal degrees of freedom of $\mathbf{u}^h$ associated to node I, respectively.

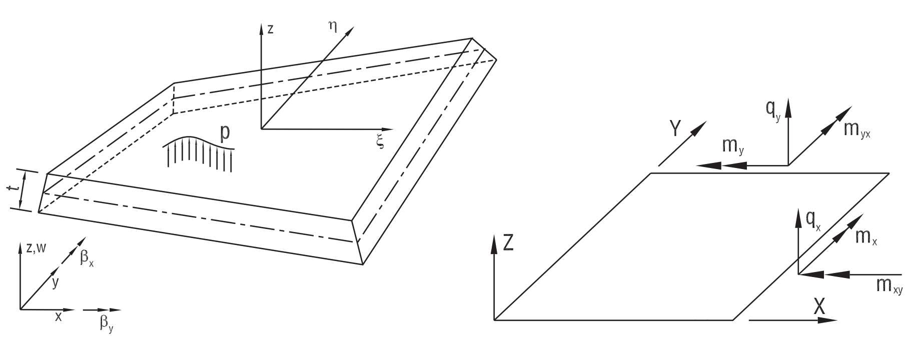

Fig. 1. Geometry of a typical Mindlin–Reissner plate.