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Adaptive mesh coarsening for quadrilateral and hexahedral meshes

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ABSTRACT

Mesh adaptation methods can improve the efficiency and accuracy of solutions to computational modeling problems. In many applications involving quadrilateral and hexahedral meshes, local modifications which maintain the original element type are desired. For triangle and tetrahedral meshes, effective refinement and coarsening methods that satisfy these criteria are available. Refinement methods for quadrilateral and hexahedral meshes are also available. However, due to the added complexity of maintaining and satisfying constraints in quadrilateral and hexahedral mesh topology, little research has occurred in the area of coarsening or simplification. This paper presents methods to locally coarsen conforming all-quadrilateral and all-hexahedral meshes. The methods presented provide coarsening while maintaining conforming all-quadrilateral and all-hexahedral meshes. Additionally, the coarsening is not dependent on reversing a previous refinement. Several examples showing localized coarsening are provided.

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1. Introduction

Mesh adaptation methods can improve the efficiency and accuracy of solutions to computational modeling problems. For a given model, there are usually regions that require greater mesh density than others to improve solution efficiency, reduce error or uncertainty in high gradient regions, or more accurately represent the model geometry. Regions where high accuracy is not critical or where gradients are low can generally be modeled with lower mesh density. Since the computational time required in a finite element analysis is directly related to the number of elements in the model being analyzed, it is advantageous to produce a mesh that has as few elements as possible. Therefore, in an ideal analysis, each region in the model should have enough elements to produce a good solution, but no more.

Due to the complexity inherent in many mesh generation algorithms, it is often difficult to create an initial mesh that optimizes both accuracy and efficiency. Although some control over mesh density is possible, an initial mesh will almost always contain regions that have too few elements, regions that have too many elements, or both. In addition, some applications require mesh density to evolve throughout an analysis as areas of high and low activity change with time [1–4]. For these reasons, much research has been devoted to the development of mesh modification tools that make it possible to adjust element density in specific regions either before or during analysis.

Mesh adaptation consists of both refinement and coarsening. Refinement is the process of adding elements to a mesh while coarsening is the process of removing elements from a mesh. By refining areas that have too few elements and coarsening areas that have too many elements, a more accurate and efficient analysis can be performed. Mesh adaptation methods are also useful in visual applications where objects far from view can be highly simplified while objects closer to view should have more detail. Because computer visualizations are typically embedded on a mesh, efficient algorithms for mesh adaptation are valuable for improving memory performance for views consisting of large numbers of mesh elements.

To date, most of the research in mesh adaptation has focused on refinement techniques for increasing local element density [5,6]. Complementary algorithms for decreasing local element density by element removal (i.e. coarsening) could be a powerful companion tool to refinement algorithms, potentially allowing more flexible mesh adaptation. For example, given a uniform mesh, the mesh density in an area of interest may be increased by established refinement techniques and decreased away from the areas of interest using a coarsening technique. Rather than remeshing the model, the

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base mesh may be modified using refinement and coarsening tools; this would permit increased resolution and accuracy in the results while maintaining a similar computation time for the entire model. Furthermore, a given model may require adaptation in different locations depending on different load cases, adaptation by both refinement and coarsening from a single base mesh may allow more efficient and robust generation of meshes appropriate for varied circumstances. In spite of its potential benefits, coarsening is an area of research which has received limited attention.

In this paper we describe algorithms for performing coarsening on all-quadrilateral and all-hexahedral meshes while maintaining conforming mesh topology through the coarsening process to prevent the creation of non-quadrilateral or non-hexahedral elements. In the following sections we will discuss related work in mesh coarsening or simplification, outline our algorithms and demonstrate the algorithms on several examples.

2. Background

Mesh adaptation is a field which has received extensive study among both computational mechanics and computer graphics researchers. Generally these two fields have not collaborated due to the many additional restrictions associated with computational mechanics but unnecessary in computer graphics. One example of these additional restrictions in computational mechanics is that a mesh must accurately represent the model geometry by ensuring that the nodes representing a curve or surface of the model do not move off the geometry, whereas in graphics a sufficiently low level of detail might justify combining surfaces and/or curves.

To effectively achieve the objectives of mesh adaptation, a truly general quadrilateral/hexahedral coarsening algorithm should:

1. Preserve a conforming all-hexahedral or all-quadrilateral mesh.
2. Restrict mesh topology and density changes to defined regions.
3. Work on both structured and unstructured meshes.
4. Not be limited to only undoing previous refinement.

2.1. Triangle and quadrilateral simplification algorithms

Triangular meshes in computer graphics and computational mechanics are common due to the relative simplicity of generating the meshes from these simplex elements. Triangle meshing algorithms are well-established and on-going efforts in the research community continue to improve the quality of these meshes. Triangle mesh simplification algorithms begin with an existing base mesh, consisting of triangles, and modify the topology to remove triangles, improve quality and/or geometric integrity. A survey of triangle mesh coarsening algorithms is documented by Cignoni et al. [7], highlighting the major simplification methodologies, including coplanar facet merging, controlled vertex/edge/face decimation, retiling, energy function optimization, vertex clustering, wavelet based approaches, and simplification via intermediate hierarchical representation. Additional surveys that compare smaller sets of algorithms are also given in [8].

One of the foremost algorithms of triangle mesh simplification was developed by Garland et al. [9]. The approach is fast, reliable, and is also generally applicable to any polygon mesh. The algorithm assumes that the mesh is composed entirely of triangles, or can be broken into a mesh composed of triangles. It is designed to combine surfaces and curves that are indistinguishable when rendered at a low level of detail. Hoppe et al. [10], demonstrate mesh adaptation respecting geometric curves and surfaces in order to preserve sharp corners and edges in the mesh representation.

While triangle meshes have widespread use, quadrilateral meshes are sometimes preferred in computational analysis due to some beneficial mathematical properties of the quadrilateral element that can result in increased solution accuracy with fewer elements than triangle meshes [11]. Unfortunately, despite the wide availability of triangle mesh adaptation algorithms, most of algorithms developed for triangle mesh simplification cannot be adapted for use on quadrilateral meshes.

A number of efforts have been utilized for quadrilateral coarsening of structured meshes. Takeuchi et al. [12], modified the approach developed by Garland et al. [9], to simplify quadrilateral meshes; however, the process is designed for full-model simplification and may produce degenerate elements (i.e. quadrilaterals which are inverted or concave). Cheng et al. [13] developed a method of coarsening a structured, all-quadrilateral mesh specifically for use on auto-body parts; however, this method has not been adapted for use in unstructured meshes. Kwak et al. [14] performs simplification using remeshing algorithms; however, this global approach can be slow when only local adaptation is needed. Choi [15] describes an algorithm which can be used to undo previous refinement on both quadrilateral and hexahedral meshes; however, the reliance on knowledge of previous refinement restricts the algorithm from being used on a base mesh that has not been refined. Nikishkov [16] developed a quadtree method for mesh adaptation that allows both refinement and coarsening; however, his method requires the use of special elements or produces non-conforming elements.

2.2. Hexahedral coarsening

Although hexahedral coarsening has been utilized in some modeling applications, no single algorithm has been developed that satisfies all the criteria listed above. This is, in large part, due to the topology constraints that exist in a conforming all-hexahedral mesh. These constraints make it difficult to modify mesh density without causing topology changes to propagate beyond the boundaries of a defined region [17,18].

Since current hexahedral coarsening methods are unable to satisfy all the requirements listed above, they have limited application. For example, to prevent global topology changes, some algorithms introduce non-conforming or non-hexahedral elements into the mesh [1,2,19–21]. While this is a valid solution for some types of analysis, not all finite element solvers can accommodate hanging nodes or hybrid meshes. Other algorithms maintain a conforming all-hexahedral mesh, but they generally require either global topology changes beyond the defined coarsening region [18,22,23], structured mesh topology where predetermined transition templates can be used [24,22], or prior refinement that can be undone [2,19,20]. These weaknesses severely limit the effectiveness of these algorithms on most real-world models.

3. Dual methods

In recent years, a greater understanding of quadrilateral and hexahedral mesh topology has led to the development of many new quadrilateral and hexahedral mesh operations [25–28]. The algorithms presented in this paper utilize the dual representation of a quadrilateral/hexahedral mesh. In this section, we discuss dual-based operations which are useful for modification of quadrilateral and hexahedral meshes.

3.1. Quadrilateral dual methods

A *dual chord* is a set of quadrilaterals connected through pairs of opposite edges that extend through a mesh to connect back at the original starting edge (on a closed surface) or terminate at the mesh boundary (for a bounded quadrilateral mesh). In Fig. 1, the dashed line highlights a single chord of the quadrilateral mesh shown.