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Sweeping of unstructured meshes over generalized extruded volumes

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ABSTRACT

An algorithm for the discretization of general extrudable volumes into semi-structured meshes is presented in this paper. The set of considered extrudable volumes is limited to volumes formed by two opposite topologically identical and geometrically similar cap surfaces, from which one is identified as the source and the other one as the target surface, and by the set of lateral quad-mappable surfaces. The source cap surface is discretized by an appropriate unstructured surface mesh which is projected onto the target cap surface using an affine mapping defined between the parametric spaces of the cap surfaces. This mapping is established by a least-squares fit of boundary nodes on the target surface. The structured quad meshes on lateral surfaces are obtained by transfinite interpolation. The volume interior nodes used to form inner layer of elements are defined by the mapping derived from a least-squares approximation of all boundary nodes. The obtained mesh is then subjected to a specific two-phase smoothing. The capabilities of the proposed algorithm are demonstrated by several examples.

1. Introduction

Algorithms for the creation of extruded meshes are generally considered very attractive and are usually implemented by most of the mesh generation packages. There are several reasons for that. First of all, prismatic elements as bricks and wedges, produced by the extrusion methods, are quite popular in the engineering analysis community (compared to the tetrahedral elements) because of their favorable features especially from the computational point of view. However, a general purpose fully automatic algorithm capable to discretize complex geometrical domains of arbitrary topology into a valid high quality hexahedral mesh is still hot topic of the research. Therefore, a lot of attention has been focused on the development of decomposition methods. These methods are based on splitting the domain volume into simple parts that can be effectively discretized by specialized algorithms. One of them is the extrusion method, which has the potential to handle quite general shapes often referred to as generalized cylinders.

The topology of a simple one-to-one extruded volume is defined by a pair of cap surfaces on its top and bottom sides which are linked together by a set of lateral surfaces. The cap surfaces must be topologically equivalent to each other and the lateral surfaces have to be quad mappable, this means bounded by four sides from which one pair of opposite sides is shared with the corresponding surface from the pair of cap surfaces. In the most simple case, the extruded shape is defined by a surface swept along a particular control curve from the initial position corresponding to the bottom (source) cap surface to the final position corresponding to the top (target) cap surface. This kind of volume can be generalized by allowing the surface to change its orientation in space while being swept. Discretization of such class of volumes might seem relatively simple because the cross-section along the whole sweeping trajectory is geometrically identical (except for some rigid body motions) with the cap surface. Thus the location of all boundary and interior points can be calculated from the cap surface geometry, the control curve shape, and the applied rigid body motions. However, problems arise when the volume starts to fold over itself.

In this work, a more complex class of extruded volumes, operating with cap surfaces that are not geometrically identical, is considered. This implies several significant features. Firstly, the cap surfaces may be of different area, shape and curvature. Moreover, there does not exist a single common control curve describing the extrusion. The sweep trajectory is uniquely defined only for those boundary vertices of the cap surfaces that correspond to appropriate sides (in the sweeping direction) of lateral surfaces.[1] The standard procedure to generate an extruded semi-structured mesh then consists of the

[1] Note that if the lateral surfaces are already discretized by structured quadrilateral mesh, then the sweeping trajectory is defined for all boundary nodes of cap surfaces.

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following steps:

1. identify the source and target cap surfaces,
2. generate an appropriate unstructured surface mesh over the source surface,
3. generate a valid surface mesh over the target surface that is topologically identical with the source mesh,
4. generate a valid structured quadrilateral mesh over lateral surfaces compatible with the cap surfaces,
5. generate a valid volume mesh inside the extruded volume.

With respect to the sweeping methodology, the most specific algorithms are related to steps 1, 3, and 5. The mesh generation in steps 2 and 4 can be performed by ordinary structured and unstructured surface mesh generation techniques. However, it is important to mention that the quality of source and lateral surface meshes has usually a great impact on the quality of the resulting volume mesh. The identification of the source and target surfaces is rather straightforward, using just topological information, if one-to-one sweeping volume is considered. However, sophisticated approaches are necessary [1–4] if multiple source to multiple target sweeping or multi-axis directional sweeping is to be applied. The generation of the valid surface mesh on the target surface, with the same connectivity as the source mesh, is not a trivial task. The problem is that cap surfaces may differ significantly in the shape and curvature. This implies that standard unstructured surface mesh generation methods cannot be generally adopted. Instead, an appropriate projection technique has to be applied. A simple and efficient projection approach based on a least-squares approximation of an affine mapping between parametric representation of the loops of boundary nodes on the source and target cap surfaces was presented in [5]. In [6], a different approach based on copying and morphing of unstructured quadrilateral meshes was elaborated. A lot of research was focused on the placement of interior nodes which is crucial for producing a high quality volume mesh. An early and rather single purpose advancing front based approach introduced in [7] was replaced by a layer by layer boundary mesh based interpolation in [8]. An elegant method using linear transformation determined by means of a least-squares fit between the loops of the boundary nodes on cap surface and boundary nodes of particular layer of extruded elements was published in [9]. In [5], this approach was modified to a weighted least-squares fit taking into account boundary nodes on both cap surfaces. Note that for many geometries (especially with non-convex cross-section or with not simply connected cap surfaces), the interior nodes have to be subjected to a layer based smoothing [9,6] in order to improve the volume mesh quality.

In the present work, a slightly different method for the mesh generation in steps 3 and 5 is adopted. It uses a least-squares approximation of appropriate mapping based on Bernstein polynomials. The mapping between the parametric spaces of source and target surfaces is approximated by a Bezier surface of appropriate order. Points of the control polygon of that surface are determined by means of a least-squares fit between the parametric coordinates of corresponding boundary nodes on the source and target surfaces. Since the intermediate layers of nodes are not described by a surface parameterization, this approach cannot be directly applied to the generation of volume interior nodes. Instead, the extruded volume is approximated by a Bezier volume of appropriate order. Its control polygon points are again determined by a least-squares fit but this time applied to all the boundary nodes of the extruded volume. Because neither the mapping defined by the Bezier surface nor the mapping defined by the Bezier volume is exact, the surface mesh on the target surface as well as the final volume mesh is subjected to appropriate smoothing.

The outline of this paper is as follows. Initially, representation of a Bezier surface and volume is recalled in Section 2. The actual sweeping algorithm is described in Section 3 in which a separate subsection is dedicated to each of the above steps for the generation of extruded meshes. The capabilities of the proposed extrusion methodology are demonstrated by a few examples in Section 4 and the paper ends with concluding remarks in Section 5.

2. Bezier surface and volume representation

Bezier surfaces are typically used for the representation of free-form geometrical entities. The Bezier surface of order $M \times N$ can be written in the form

$$\boldsymbol{r}(u,v) = \sum_{i=1}^{M}\sum_{j=1}^{N} B_i^M(u) B_j^N(v) \boldsymbol{P}_{i,j}, \tag{1}$$

where $\boldsymbol{r}(u,v)$ is the positional vector of a point on the surface, $\boldsymbol{P}_{i,j}$ are Bezier control polygon points, $B_i^M(u)$ and $B_j^N(v)$ stand for Bernstein polynomials of order M and N, and parameters u and v denote curvilinear coordinates of the surface ranging from 0 to 1. While the surface is interpolating the corner control points $\boldsymbol{P}_{1,1}$, $\boldsymbol{P}_{M,1}$, $\boldsymbol{P}_{1,N}$, and $\boldsymbol{P}_{M,N}$, the remaining control points (if any) are only approximated by the surface affecting the profile of curvilinear coordinates u and v over the surface (Fig. 1 a). Note that if all the control points $\boldsymbol{P}_{i,j}$ are coplanar, then the defined surface is planar, but generally folded over itself. Similarly, the Bezier volume (Fig. 1b) of order $M \times N \times P$ can be described by

$$\boldsymbol{r}(u,v,t) = \sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{P} B_i^M(u) B_j^N(v) B_k^P(t) \boldsymbol{P}_{i,j,k}. \tag{2}$$

Again, the region is interpolating the corner control points, the remaining control points are only approximated. Bernstein polynomials of order N can be expressed as

$$B_i^N(t) = \binom{N-1}{i-1} t^{i-1}(1-t)^{N-i}, \quad i = 1,2,\ldots,N \tag{3}$$

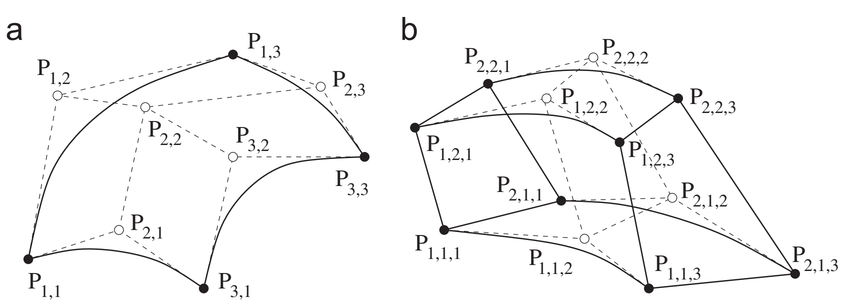

Fig. 1. Representation of Bezier entities: (a) Bezier surface (order 3 × 3), (b) Bezier volume (order 2 × 2 × 3).