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Two finite elements for general composite beams with piezoelectric actuators and sensors

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ABSTRACT

Two finite elements for the static analysis of smart beams with piezoelectric sensors/actuators are presented: ad hoc smart beam element (ADSBE) and variational asymptotic smart beam element (VASBE). Both elements rely on the computation of the cross-sectional matrices associated with the electromechanical properties of the beam cross-section. ADSBE uses the Timoshenko cross-sectional stiffness matrix computed by the VABS program, and the electric field is assumed constant across the thickness of each piezoelectric layer. Taking advantage of the cross-section discretization of the beam, all the matrices related to the electric field are also computed by performing a numerical integration using the VABS program that was extended to account for these new quantities. VASBE is based on the fully coupled Timoshenko theory for smart beams constructed using the variational asymptotic method. This theory decouples the original three-dimensional electromechanical problem to a two-dimensional electromechanical cross-sectional analysis and a one-dimensional beam analysis. The cross-sectional analysis provides a one-dimensional constitutive model for the beam analysis without a priori assumptions regarding the geometry of the cross-section, the electric field distribution, and the location of smart materials. Several examples available in the literature are used to validate the accuracy of these two new elements. The numerical results obtained using ADSBE and VASBE correlated well with other published results. For structures that are out of the limits in which one structure may be modeled as a beam, the ADSBE showed considerable errors and, therefore, should not be used. Nevertheless, VASBE was able to predict the 3D results available in the literature with an error smaller than 8%.

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1. Introduction

In the most recent decades a tremendous advance in the technology of smart structures has shown its viability and potential on numerous applications, such that space structures, aircraft, wind turbines and helicopter rotors etc. In its fundamental nature a smart structure consists of active materials, such as piezoelectrics, which are able of sensing and reacting to external stimuli and, usually, are integrated into the structure with a control unity.

Several mathematical models have been developed to describe the behavior of structures that are actuated and sensed by piezoelectric materials. However, the analytical predictive capabilities for smart structures are still very limited in comparison to those for conventional composite structures [1]. Searches for analytical solutions lead to the simultaneous solution of the electric and mechanical equilibrium equations, which must be solved for a set of boundary conditions [2]. The coupling of the electric and mechanical constitutive equations will lead to the coupling of some boundary conditions, thus the use of the conventional mechanical boundary conditions may not be adequate to accurately predict the electromechanical boundary coupling [3]. Exact solutions exist for a very few specialized and idealized cases. For general cases, one needs to use the finite element method to solve the coupled electromechanical systems.

Structures can be analyzed using beam models if one dimension is much larger than the other two dimensions. To take advantage of this geometrical future, several researchers have proposed various smart beam models to capture the behavior associated with the two small dimensions eliminated in the one-dimensional beam analysis [1]. This work handles two of the three models most used, the engineering model that is based on a priori kinematic assumptions and the asymptotic model which is based on the asymptotic expansion of the three-dimensional quantities. Due to the huge number of works that have been proposed, the engineering models dominate the literature on the modeling of smart beams. An extensive and

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comprehensive review on the existing development and ongoing progress in smart structures can be found in Refs. [4,5]. They can be further classified as uncoupled models if only the induced strain effect is modeled [6–9], or coupled models if both actuation and sensing capabilities of smart materials are modeled simultaneously [10–15].

The first order deformation theory which includes the Timoshenko theory for smart beams with actuation and sensing capabilities implicitly incorporated in the model is used in this work. Since real applications may involve actuators and sensors distributed within the composite material of the beam cross-section, this formulation will utilize the constitutive relations computed by VABS to model the mechanical behavior. The voltage distribution within any piezoelectric layer is assumed to be linear and the electric potential distribution is modeled by the discrete linear layerwise formulation. The resulting matrices of the layerwise formulation are computed numerically by extending the VABS routines to these news quantities.

The variational asymptotic method (VAM) introduced by Berdichevsky [16] has been used to construct models with both merits of engineering models, related with a systematic and easy implementation, and asymptotic models characterized by without invoking a priori kinematic assumptions [17]. VAM was, recently, applied to develop classical models for smart thin-walled beams and solid beams and refined models for smart solid beams [18–20]. More recently, Roy et al. [1] presented an asymptotically correct classical and Timoshenko beam model for smart beams [21]. No assumptions were made on the distribution of mechanical and electric field inside the structure.

In this study, we will first develop an ad hoc smart beam element (ADSBE) based on the first order deformation theory, where the mechanical displacement and the electric potential along the beam axes is approximated by quadratic Lagrangean shape functions. Then, based on the work by Roy [21], we will develop a variational asymptotic smart beam element (VASBE) so that the actuating and sensing capabilities can be implicitly included. The performance of both elements is compared with several other models available in the literature.

2. Ad hoc smart beam element (ADSBE)

The mathematical formulation of ADSBE uses a first order displacement field and a piecewise, linear through-the-thickness electric potential.

2.1. Variational principle

This formulation will be based on the principle of virtual work in which the potential energy accounts for the parts of the structure that are made with piezoelectric material. The general form of this principle is stated as

$$\int_{\Omega} \delta(U - W)\,\mathrm{d}\Omega = 0 \tag{1}$$

where U represents the potential energy of the flexible body and W the potential of the applied forces that are acting in the body.

The total potential energy for a structure that has parts made with piezoelectric materials is called electric enthalpy and is given as [10]

$$U = H = \frac{1}{2}\int_{\Omega} (\boldsymbol{\varepsilon}^{\mathrm{T}}\mathbf{C}^{E}\boldsymbol{\varepsilon} - 2\mathbf{E}^{\mathrm{T}}\mathbf{e}\boldsymbol{\varepsilon} - \mathbf{E}^{\mathrm{T}}\mathbf{g}\mathbf{E})\,\mathrm{d}\Omega \tag{2}$$

where $\boldsymbol{\varepsilon}$ is the strain vector, $\mathbf{E}$ is the electric field, $\mathbf{C}^{E}$ is the elasticity tensor at constant electric field, $\mathbf{e}$ is the piezoelectric tensor, and $\mathbf{g}$ is the dielectric tensor at constant strain. Furthermore, the magnetically static electric field $\mathbf{E}$ is related to the electric potential field Φ as [10]

$$\mathbf{E} = -\nabla\Phi \tag{3}$$

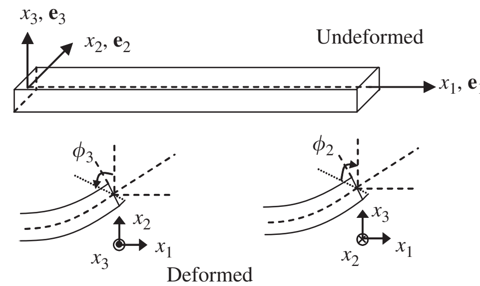

Fig. 1. Slope and cross-sectional rotation of the beam.

The total work W done by the external mechanical and electrical loading is given by [22]

$$W = \int_{\Omega} \mathbf{u}^{\mathrm{T}}\mathbf{f}_b\,\mathrm{d}\Omega + \int_{\Gamma_s} \mathbf{u}^{\mathrm{T}}\mathbf{f}_s\,\mathrm{d}\Gamma_s - \int_{\Gamma_\varphi} \varphi q_0\,\mathrm{d}\Gamma_\varphi + \sum_i \mathbf{u}_i^{\mathrm{T}}\mathbf{f}_i^c \tag{4}$$

in which $\mathbf{u}$ is the displacement field, $\mathbf{f}_b$ is the body load vector, $\mathbf{f}_s$ is the surface load vector and $\mathbf{f}_i^c$ is the ith concentrated load vector. The charge density is represented by q_0 and Γ_s, Γ_φ are used to represent the surfaces that are loaded mechanically and electrically, respectively. Substituting Eqs. (2)–(4) into Eq. (1) and computing its variation yields to the following variational statement:

$$\begin{aligned}&\int_{\Omega} (\delta\boldsymbol{\varepsilon}^{\mathrm{T}}\mathbf{C}^{E}\boldsymbol{\varepsilon} + \delta\boldsymbol{\varepsilon}^{\mathrm{T}}\mathbf{e}^{\mathrm{T}}\nabla\Phi + (\nabla\delta\Phi)^{\mathrm{T}}\mathbf{e}\boldsymbol{\varepsilon} - (\nabla\delta\Phi)^{\mathrm{T}}\mathbf{g}\nabla\Phi - \delta\mathbf{u}^{\mathrm{T}}\mathbf{f}_b)\,\mathrm{d}\Omega\\ &\quad - \int_{\Gamma_s} \delta\mathbf{u}^{\mathrm{T}}\mathbf{f}_s\,\mathrm{d}\Gamma_s + \int_{\Gamma_\varphi} q_0\delta\Phi\,\mathrm{d}\Gamma_\varphi - \sum_i \delta\mathbf{u}^{\mathrm{T}}\mathbf{f}_i^c = 0\end{aligned} \tag{5}$$

Because the strain field is related to displacement field, in Eq. (5), there are four unknowns including the electric potential φ and the mechanical displacements $\mathbf{u}$. The solution of these unknowns will be obtained numerically using the finite element method.

2.2. Mechanical displacement and strain

Consider a set of unit vectors $\mathbf{e}_1$, $\mathbf{e}_2$, and $\mathbf{e}_3$ attached to the centroid of the beam cross-section, with $\mathbf{e}_1$ aligned with the beam axis while $\mathbf{e}_2$ and $\mathbf{e}_3$ used to define cross-sectional plane, as it is illustrated in Fig. 1.

Let $u_1(x_1,x_2,x_3)$, $u_2(x_1,x_2,x_3)$ and $u_3(x_1,x_2,x_3)$ be the displacement components of an arbitrary point of the beam in the $\mathbf{e}_1$, $\mathbf{e}_2$, and $\mathbf{e}_3$ directions, respectively. If the cross-section is assumed to move like a rigid body, the displacement field in the plane of the cross-section is

$$\begin{aligned}u_1(x_1,x_2,x_3,t) &= u_1^0(x_1,t) + x_3\phi_2(x_1,t) - x_2\phi_3(x_1,t)\\ u_2(x_1,x_2,x_3,t) &= u_2^0(x_1,t) - x_3\phi_1(x_1,t)\\ u_3(x_1,x_2,x_3,t) &= u_3^0(x_1,t) + x_2\phi_1(x_1,t)\end{aligned} \tag{6}$$

where u_1^0, u_2^0, u_3^0 are the displacements of the neutral axes and ϕ_1 is the twist angle of the cross-section about x_1. The rotations of the cross-section $\phi_2(x_1,t)$ and $\phi_3(x_1,t)$ are positive about axes $\mathbf{e}_2$ and $\mathbf{e}_3$, respectively. The set of strain equations can be derived from

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