Finite Elements in Analysis and Design 45 (2009) 305–314

Contents lists available at ScienceDirect

Finite Elements in Analysis and Design

journal homepage: www.elsevier.com/locate/finel

A family of interface elements for the analysis of composite beams with interlayer slip

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A R T I C L E I N F O

Article history:
Received 8 May 2008
Received in revised form 19 September 2008
Accepted 13 October 2008
Available online 5 December 2008

Keywords:
Composite beams
Partial interaction
Interlayer slip
Interface elements

A B S T R A C T

This work presents the numerical formulation of a family of zero-thickness interface elements developed for the simulation of composite beams with horizontal deformable connection, or interlayer slip. The proposed elements include formulations to be employed with Euler–Bernoulli as well as with Timoshenko beam theories, combined to displacement-based beam elements sharing the same degrees of freedom. The elements may be employed for the simulation of steel–concrete composite beams, layered beams or other structural systems in which components are connected with the possibility of interlayer slip. Numerical examples are provided to assess the accuracy and robustness of the formulations and to identify the most reliable formulations. Abnormal slip distributions are shown to occur also with the Timoshenko-based elements, usually in conjunction with transverse shear locking.

1. Introduction

Composite beams are usually made of a steel profile linked to a concrete slab by some sort of shear connection, which may allow for relative tangential displacements between the elements. This effect, also called partial interaction, plays an important role in the analysis and design of such structures. A very similar behavior may show up in several other structural systems, such as precast concrete slabs, wood–concrete floor systems, sandwich panels and multilayered wood beams, either glued to each other or connected by some mechanical device.

The problem of the layered beam with deformable connection has been studied for a long time. The first solutions to the problem involved development of analytical models, such as the much quoted Newmark differential equation [1]. This analytical formulation addresses beams with two linear elastic materials and assumes identical vertical displacements, rotations and curvatures for the connected members.

Newmark's model was later extended for the case of vertical partial interaction by Adekola [2]. The governing system of differential equations was derived based on equilibrium, and the unknowns included the vertical force per unit length and the shear flow at the interface, as well as the moment distribution along the beam length. An exact solution for beam-columns with interlayer slip was developed by Girhammar and Gopu [3], where the composite beam was subjected to normal forces and transverse loading.

Analytical solutions provided by differential equations remain an essential resource for the assessment of numerical results. These are usually based on Euler–Bernoulli (EB) assumptions, namely, cross sections remain plane and normal to the deformed axis after deformation. Exact solutions without beam theory assumptions are difficult to obtain, but recently, Xu and Wu [4] provided analytical solutions to simply supported beams with interlayer slip, based on a plane stress model.

The impossibility of finding exact solutions for general cases, including continuity, material nonlinearity and other phenomena, and the need to consider partial interaction behavior in practice led to the development of numerical solutions. Among these, the finite element method (FEM) appears to be the most successful, due to its versatility, robustness and sound mathematical basis.

A brief review of recent work on composite beams with partial interaction follows. Dall'Asta and Zona [5] developed a family of displacement-based elements for the analysis of partially connected beams, which later was extended with the development of three-field mixed elements [6]. Mixed or force-based elements were the subject of various works [7–9]. A critical review of some numerical techniques has been carried out by Ranzi et al. [10]. In the past few years, some researchers have employed analytical results to develop partial interaction formulations [11–13].

Most of the numerical research on partially connected beams has been based on EB beam theory. Nonetheless, recent works have focused on elements based on Timoshenko theory. Ranzi and Zona [14] presented analytical and FE formulations for steel–concrete composite beams taking into account the shear deformability of the steel component, along with an extensive parametric study, for short- and

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doi:10.1016/j.finel.2008.10.007

long-term loading. According to the authors, the shear deformability was restricted to the steel element due to the fact that nonlinear concrete models for beams are usually given in terms of uniaxial stress states. They concluded that shear effects need to be carefully evaluated as the results may differ considerably from the EB results, especially with continuous systems.

Considering that some beam elements may exhibit shear locking, Schnabl et al. [15] developed a strain-based finite element for the simulation of two-layer systems, employing a modified principle of virtual work and a Petrov–Galerkin collocation method.

Shear-deformable beam theory is particularly suited for problems where the span/depth ratio is not large, and the magnitude of shear stresses is considerable. Moreover, provided that shear locking is prevented, it can also model slender beams. Due to this generality, Timoshenko-based FE formulations for partially connected composite beams are attractive.

The purpose of this paper is to present alternative numerical solutions for the analysis of composite beams with deformable shear connection, or interlayer slip. Following a previously developed EB-based zero-thickness interface element [16], other elements are introduced. The first of them is also EB-based, but with an extra degree of freedom on the axial displacement interpolation to prevent slip locking. Three other interface elements, based on Timoshenko beam theory and differing in their interpolation schemes, are developed and combined with the usual displacement-based beam elements and, in order to alleviate locking effects, with assumed strain beam elements. The merits and shortcomings of each formulation are discussed by means of numerical examples and comparison with analytical solutions.

2. Kinematical model

The kinematical hypotheses underlying partially connected beams with horizontal partial interaction may be summarized as follows. Assuming plane sections remain plane, and that the composite beam lies in the xy plane, the displacement field is given by (Fig. 1)

$$u_\alpha(x,y) = u_\alpha^0(x) + (y - y_\alpha)\theta(x) \tag{1}$$

$$v_\alpha(x,y) = v^0(x) \tag{2}$$

where $\alpha = 1 \ldots 2$ represents the upper and the lower element, respectively, and y_α is the reference axis for each constituent element, which is usually taken at the centroid.

From the displacement field and adopting suitable interpolation schemes the FE formulation is derived, usually by the virtual work principle.

3. Interface elements

The finite element solution to problems in which adjacent elements have different tangential displacements has been investigated by engineers and researchers for a long time. Interface elements must be able to predict and allow slip and debonding between two bodies in contact, or separated by a thin material layer. Constitutive relations may include slip, nonslip, separation and rebonding. In some cases, special formulations and contact detection algorithms must be employed.

The simulation of an interface zone can be performed by means of (a) thin continuum finite elements, (b) linkage elements in which opposite nodes are connected by discrete springs or (c) interface elements with zero or finite thickness.

Thin continuum finite elements have been employed for geotechnical problems. Their main advantage is that no new element formulation is introduced, but they present numerical ill-conditioning for low thickness/length ratios.

Linkage elements for interface problems were first employed by Herrman [17]. Their formulation is quite simple as springs are attached to the element nodes in order to simulate the relative displacement. Some formulations for the analysis of composite beams employ this kind of strategy. Gattesco [18] developed a numerical solution for the nonlinear analysis of composite beams by attaching springs to the element ends. More recently, this type of approach was employed by Ranzi et al. [13] and Gara et al. [20] on the simulation of composite beams with combined longitudinal and transverse partial interaction.

The element developed by Goodman, Taylor and Brekke [21] (also known as GTB) was the first zero-thickness interface element. It enables the simulation of relative displacements within two adjacent finite elements, by the independent interpolation of the relative displacements on opposite edges. The strains are then evaluated from these relative displacements, and from the expression for the strain energy the element stiffness matrix may be obtained. The stiffness expression contains values of the element height h, so that small values must be used to simulate the 'zero' thickness. Extensions of the original GTB element were later proposed for the simulation of 3D problems [22,23].

The interface element formulation proposed here is derived from the original zero-thickness GTB element, with displacement fields on the top and bottom edges consistent with the related upper and lower beam elements (Fig. 2). The displacement field has for the tangential relative movement

$$w_h(x) = u_2^0(x) - u_1^0(x) + (y_2 - d)\theta_2(x) - (y_1 - d)\theta_1(x) \tag{3}$$

and for the normal relative displacement:

$$w_v(x) = v_2(x) - v_1(x) \tag{4}$$

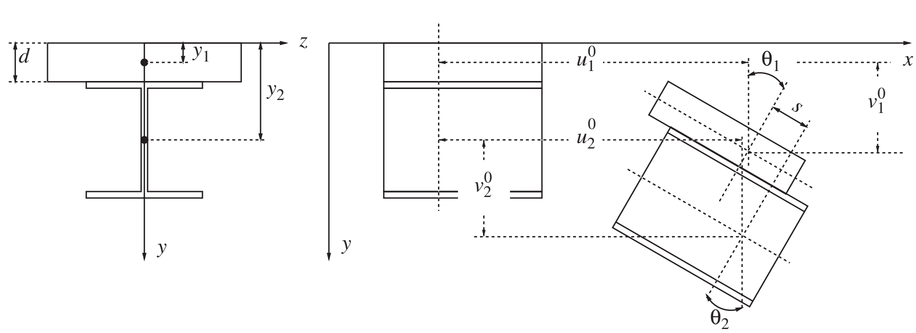

Fig. 1. Displacement field of composite beam.