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Design and optimization of three-dimensional extrusion dies, using constraint optimization algorithm

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ABSTRACT

Balancing the distribution of flow through a die to achieve a uniform velocity distribution is the primary objective and one of the most difficult tasks of extrusion die design. If the manifold in a Coat-hanger die is not properly designed, the exit velocity distribution may be not uniform; this can affect the thickness across the width of the die. Yet, no procedure is known to optimize the coat hanger die with respect to an even velocity profile at the exit. While optimizing the exit velocity distribution, the constraint optimization algorithm used in this work enforced a limit on the maximum allowable pressure drop in the die; according to this constraint we can control the pressure in the die. The computational approach incorporates three-dimensional finite element simulations software Rem3D® and includes an optimization algorithm based on the global response surfaces with the Kriging interpolation and SQP algorithm within an adaptive strategy of the search space to allow the location of the global optimum with a fast convergence. The optimization results which represent the best die design are presented according to the imposed constraint on the pressure.

1. Introduction

The design of dies for polymer extrusion often involves trial and error corrections of the die geometry to achieve uniform flow at the exit. If the repartition channel in a flat die is not designed properly, the velocity at the exit of the flat die may not be uniform [1], and leads to a variation in the sheet thickness across the width of the die.

Often, the number of the involved variables and their interactions prevent any optimization according to the trial and error corrections, because the number of evaluations needed may become very high. Design of experiment, in particular the Taguchi method [2], allows obtaining invaluable information on the important variables of the process in order to achieve the required goals. The effects of the various factors can be represented on graphs to support the discussion and to lead to identify the most sensitive to minimize the defects. Within this framework, we can mention Chen et al. [3]. They showed, using the Taguchi method, that the operating conditions, the type of materials, and the geometry of the die have a great influence on the exit velocity distribution on the die.

Prior works in sheet die optimization have involved the use of lubrication approximations of the momentum equations [4,5]. If the geometry is more complex, a flow channel can be approximated with simple geometric sections [6]. Smith et al. [7,8] modeled Newtonian and non-Newtonian isothermal flow in a coat hanger die using a generalized Hele–Shaw (HS) approximation, and optimized the die by minimizing pressure drop subject to exit flow uniformity being within a tolerance set. The sensitivities analysis needed for the sequential quadratic programming (SQP) algorithm was calculated by direct differentiation and the adjoint method are compared, and simultaneous minimization of velocity dispersion subject to residence time variations is added using Broyden Fletcher Goldfarb Shanno (BFGS) algorithm and penalty function. The same author [9] in order to optimize the shape of the extrusion die for two different materials at various temperatures, used and compared two optimization algorithms with constraint based on SQP and sequential linear programming (SLP). The optimization problem consists in minimizing the pressure loss in the die, with an imposed constraint so that a homogeneous velocity distribution is obtained on the outlet side of the die within an imposed tolerance. Network algorithms have been developed to optimize die designs [10] but they are difficult to apply to arbitrary shapes. Michaeli et al. [11] have used a combination of finite element analysis for isothermal flow and flow analysis network to accelerate the iterative optimization process for the design of profile extrusion dies. To optimize the die geometry, they used, respectively, the evolution strategy algorithm and network theory. Sun et al. [12] optimize a flat die using BFGS algorithm. A penalty

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Nomenclature

a, m, τ, A_1, A_2	material constants
$\hat{a}\beta$	coefficients vectors
α	weight coefficient
A, B, C, D	optimization variables
c	dilation parameter
d_i	distance from a discrete node x_i to a sampling point x
E	velocity dispersion
E_0	initial velocity dispersion
$\varepsilon(v)$	strain rate tensor
$\tilde{\varepsilon}$	tolerance
$F(x)$	responses from the function
g	constraint function
$\dot{\bar{\gamma}}$	shear rate
η	fluid viscosity (dependent of the temperature T, pressure p, and of the strain rate tensor $\varepsilon(v)$ through the shear rate $\dot{\bar{\gamma}}$)
$\eta_0(T)$	thermal dependency
J	normalized objective function
$\tilde{J}(x)$	objective or constraint interpolate function
k	number of the basis function in regression model
N	number of nodes at the die exit
P	pressure
P_0	initial pressure in the die
$\hat{p}(x)$	basis function
R	correlation matrix
r_w	radius of support domain
$S1$, $S2$, $S3$	surfaces
T	temperature
T_{ref}	references temperature
v	velocity
v_i	exit velocity
$\bar{v}$	average exit velocity
$w_i(x)$	weight function of Gaussian type
x	design variables
$Z(x)$	random fluctuation

function was introduced to enforce a limit on the maximum allowable pressure drop in the die.

The optimization algorithm must be carefully chosen when one single analysis using three-dimensional software requires several hours of CPU time. Non-deterministic or stochastic methods such as Monte Carlo method and genetic algorithm [13] can obtain global minimum but they need a lot of evaluations for the functions to converge. Gradient methods [7–10,12,14] require the computations of the gradients of the functions; the computation of gradients by finite difference is time consuming and depends on the perturbed parameters. For the above reasons we decided to chose a response surface method (RSM) [15].

The ultimate goal of this work is to optimize the coat hanger sheet die geometry (Fig. 1) in a way that a uniform velocity distribution is obtained at the die exit with an imposed nonlinear constraint so that the pressure loss in the die must decrease compared to the initial die.

For this end, we developed an automatic optimization algorithm based on SQP algorithm and a RSM together with Kriging interpolation and several strategies to permit to obtain a precise global optimum with a fast convergence. A preliminary study based on Taguchi's design of experiments method [3] was conducted, in order to identify the most sensitive design variables. To compute a 3D flow in extrusion dies we used FEM software (REM3D®) [16]. This software takes into account strain rate and temperature dependence.

2. Modeling and simulation

The extrusion simulation is carried out using the 3D computation software by finite elements REM3D® [16].

The flow equations are derived from the Navier–Stokes incompressible equations. A mixed finite element method for incompressible viscous flow is used. The flow solver uses tetrahedral elements with a linear continuous interpolation of both the pressure and the velocity and a bubble enrichment of velocity.

The mass, momentum and energy conservation equations, are used to follow the material behavior, from which the velocity, pressure and temperature fields are determined.

$$\begin{cases} \nabla(2\eta(\dot{\bar{\gamma}})\dot{\varepsilon}(v)) - \nabla p = 0 \\ \nabla \cdot \vec{v} = 0 \\ \rho\hat{c}\dfrac{\mathrm{d}T}{\mathrm{d}t} = -\nabla \cdot q + \sigma : \dot{\varepsilon}(v) \end{cases} \tag{1}$$

The behaviors laws used in Rem3D® give an expression of the viscosity in function of the shear rate and temperature. In this paper, the geometry of a flat die is optimized for an acrylonitrile butadiene styrene (ABS, Astalac EPC 10000). The rheological parameters of the ABS are given in Table 1. Carreau Yasuda/WLF viscosity model is used to characterize the temperature and shear rate dependence [17]. It is written as

$$\eta = \eta_0(T)\left[1 + \left(\eta_0(T)\frac{\dot{\bar{\gamma}}}{\tau_s}\right)^{\alpha}\right]^{m-1/\alpha} \tag{2}$$

In this model, a, m, τ are material constants, whereas $\eta_0(T)$ establishes the thermal dependency, given by the WLF model:

$$\eta_0(T) = \eta_0(T_{ref})\,\exp\left[\frac{A_1(T_{ref} - T_s)}{A_2 + (T_{ref} - T_s)} - \frac{A_1(T - T_s)}{A_2 + (T - T_s)}\right] \tag{3}$$

where A_1, A_2 are material constants, and T_{ref} is the references temperature.

A flow of 50 000 mm^3/s was imposed on the entry with a temperature of 240 °C, and the temperature of the die is constant and equals to 230 °C.

3. Formulation of the optimization problem

3.1. Objective and constraint functions

This optimization problem consists in determining an optimal geometry to homogenize the velocity distribution through the die exit, which corresponds to the minimum of the velocity dispersion (E). While preventing that the pressure "pressure loss" increases more than the pressure obtained by the initial geometry. We can also impose a more severe constraint on the pressure; this condition is translated by a constraint function (g).

$$\begin{cases} \min \quad J(\Phi) = \dfrac{E}{E_0} \\ \text{such that} \quad g = \dfrac{P - (\alpha * P_0)}{(\alpha * P_0)} \leqslant 0 \end{cases} \tag{4}$$

where (J), the normalized objective function, the velocity dispersion (E), is defined as

$$E = \left(\frac{1}{N}\sum_{i=1}^{N}\left(\frac{|v_i - \bar{v}|}{\bar{v}}\right)\right) \tag{5}$$