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An improved, fully symmetric, finite-strain phenomenological constitutive model for shape memory alloys

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ABSTRACT

The ever increasing number of shape memory alloy applications has motivated the development of appropriate constitutive models taking into account large rotations and moderate or finite strains. Up to now proposed finite-strain constitutive models usually contain an asymmetric tensor in the definition of the limit (yield) function. To this end, in the present work, we propose an improved alternative constitutive model in which all quantities are symmetric. To conserve the volume during inelastic deformation, an exponential mapping is used to arrive at the time-discrete form of the evolution equation. Such a symmetric model simplifies the constitutive relations and as a result of less nonlinearity in the equations to be solved, numerical efficiency increases. Implementing the proposed constitutive model within a user-defined subroutine UMAT in the nonlinear finite element software ABAQUS/Standard, we solve different boundary value problems. Comparing the solution CPU times for symmetric and asymmetric cases, we show the effectiveness of the proposed constitutive model as well as of the solution algorithm. The presented procedure can also be used for other finite-strain constitutive models in plasticity and shape memory alloy constitutive modeling.

1. Introduction

Shape memory alloys (SMAs) are unique materials with the ability to undergo large deformations regaining the original shape either during unloading (superelastic or pseudo-elastic (PE) effect) or through a thermal cycle (shape-memory effect (SME)) [1,2]. Since such effects are in general not present in standard alloys, SMAs are often used in innovative applications. For example, nowadays pseudo-elastic Nitinol is a common and well-known engineering material in the medical industry [3,4].

The origin of SMA features is a reversible thermo-elastic martensitic phase transformation between a high symmetry, austenitic phase and a low symmetry, martensitic phase. Austenite is a solid phase, usually characterized by a body-centered cubic crystallographic structure, which transforms into martensite by means of a lattice shearing mechanism. When the transformation is driven by a temperature decrease, martensite variants compensate each other, resulting in no macroscopic deformation. However, when the transformation is driven by the application of a load, specific martensite variants, favorable to the applied stress, are preferentially formed, exhibiting a macroscopic shape change in the direction of the applied stress. Upon unloading or heating, this shape change disappears through the reversible conversion of the martensite variants into the parent phase [1,5].

For a stress-free SMA material, four characteristic temperatures can be identified, defined as the starting and finishing temperatures during forward transformation (austenite to martensite), M_s and M_f, and as the starting and finishing temperatures during reverse transformation, A_s and A_f. Accordingly, in a stress-free condition, at a temperature above A_f, only the austenitic phase is stable, while at a temperature below M_f, only the martensitic phase is stable. As a consequence, applying a stress at a temperature above A_f, SMAs exhibit a pseudo-elastic behavior with a full recovery of inelastic strain upon unloading, while at a temperature below M_s, the material presents the shape-memory effect with permanent inelastic strains upon unloading which may be recovered by heating.

In most applications, SMAs experience a general thermo-mechanical loading conditions more complicated than uniaxial or

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multiaxial proportional loadings, typically undergoing very large rotations and moderate strains (i.e., in the range of 10% for polycrystals [1]). For example, with reference to biomedical applications, stent structures are usually designed to significantly reduce their diameter during the insertion into a catheter; thereby, large rotations combined with moderate strains occur and the use of a finite deformation scheme is preferred.

The majority of the currently available 3D macroscopic constitutive models for SMAs has been developed in the small deformation regime (see, e.g., [6–16] among others). Finite deformation SMA constitutive models proposed in the literature have been mainly developed by extending small strain models. The approach in most cases is based on the multiplicative decomposition of the deformation gradient into an elastic and an inelastic or transformation part [17–25], though there are some models which have utilized an additive decomposition of the strain rate tensor into an elastic and an inelastic part [26].

In the present work we focus on a finite-strain extension of the small-strain constitutive model initially proposed by Souza et al. [8] and extensively studied in Refs. [27–29]. We first develop a finite-strain constitutive model containing an asymmetric tensor, also observed in the constitutive equations of [20–24]. To this end, we propose an improved alternative constitutive model which is expressed in terms of symmetric tensors only. We then implement the proposed model in a user-defined subroutine (UMAT) in the nonlinear finite element software ABAQUS/Standard and compare the solution CPU times for different boundary value problems. The results show the increased computational efficiency (in terms of solution CPU time), when the proposed alternative symmetric form is used. This is mostly due to the simplification in computing fourth-order tensors appearing when a tensorial equation is linearized.

The structure of the paper is as follows. In Section 2, based on a multiplicative decomposition of the deformation gradient into elastic and transformation parts, we present the time-continuous finite-strain constitutive model. In Section 3, we propose an alternative constitutive equation which includes only symmetric tensors. In Section 4, based on an exponential mapping, the time-discrete form and the solution algorithm are discussed. In Section 5, implementing the proposed integration algorithm within the commercial nonlinear finite element software ABAQUS/Standard, we simulate different boundary value problems. We finally draw conclusions in Section 6.

2. A 3D finite-strain SMA constitutive model: time-continuous frame

We use a multiplicative decomposition of the deformation gradient and present a thermodynamically consistent finite strain constitutive model as done by Reese and Christ [20,21], Evangelista et al. [22] and, more recently, by Arghavani et al. [23,24]. The finite-strain constitutive model takes its origin from the small-strain constitutive model proposed by Souza et al. [8] and improved and discussed by Auricchio and Petrini [27–29].

2.1. Constitutive model development

Considering a deformable body, we denote with $\boldsymbol{F}$ the deformation gradient and with J its determinant, supposed to be positive. The tensor $\boldsymbol{F}$ can be uniquely decomposed as

$$\boldsymbol{F}=\boldsymbol{R}\boldsymbol{U}=\boldsymbol{V}\boldsymbol{R} \tag{1}$$

where $\boldsymbol{U}$ and $\boldsymbol{V}$ are the right and left stretch tensors, respectively, both positive definite and symmetric, while $\boldsymbol{R}$ is a proper orthogonal rotation tensor. The right and left Cauchy–Green deformation tensors are then, respectively, defined as

$$\boldsymbol{C}=\boldsymbol{F}^T\boldsymbol{F}, \quad \boldsymbol{b}=\boldsymbol{F}\boldsymbol{F}^T \tag{2}$$

and the Green–Lagrange strain tensor, $\boldsymbol{E}$, reads as

$$\boldsymbol{E}=\frac{\boldsymbol{C}-\boldsymbol{1}}{2} \tag{3}$$

where $\boldsymbol{1}$ is the second-order identity tensor. Moreover, the velocity gradient tensor $\boldsymbol{l}$ is given as

$$\boldsymbol{l}=\dot{\boldsymbol{F}}\boldsymbol{F}^{-1} \tag{4}$$

The symmetric and anti-symmetric parts of $\boldsymbol{l}$ supply the strain rate tensor $\boldsymbol{d}$ and the vorticity tensor $\boldsymbol{w}$, i.e.,

$$\boldsymbol{d}=\tfrac{1}{2}(\boldsymbol{l}+\boldsymbol{l}^T), \quad \boldsymbol{w}=\tfrac{1}{2}(\boldsymbol{l}-\boldsymbol{l}^T) \tag{5}$$

Taking the time derivative of Eq. (3) and using (4) and (5), it can be shown that

$$\dot{\boldsymbol{E}}=\boldsymbol{F}^T\boldsymbol{d}\boldsymbol{F} \tag{6}$$

Following a well-established approach adopted in plasticity [30,31] and already used for SMAs [17–24], we assume a local multiplicative decomposition of the deformation gradient into an elastic part $\boldsymbol{F}^e$, defined with respect to an intermediate configuration, and a transformation one $\boldsymbol{F}^t$, defined with respect to the reference configuration. Accordingly,

$$\boldsymbol{F}=\boldsymbol{F}^e\boldsymbol{F}^t \tag{7}$$

Since experimental evidences indicate that the transformation flow is nearly isochoric, we impose $\det(\boldsymbol{F}^t)=1$, which after taking the time derivative results in

$$\mathrm{tr}(\boldsymbol{d}^t)=0 \tag{8}$$

We define $\boldsymbol{C}^e=\boldsymbol{F}^{e^T}\boldsymbol{F}^e$ and $\boldsymbol{C}^t=\boldsymbol{F}^{t^T}\boldsymbol{F}^t$ as the elastic and the transformation right Cauchy–Green deformation tensors, respectively, and using definitions (2) and (7), we obtain

$$\boldsymbol{C}^e=\boldsymbol{F}^{t^{-T}}\boldsymbol{C}\boldsymbol{F}^{t^{-1}} \tag{9}$$

To satisfy the principle of material objectivity, the Helmholtz free energy has to depend on $\boldsymbol{F}^e$ only through the elastic right Cauchy–Green deformation tensor; it is moreover assumed to be a function of the transformation right Cauchy–Green deformation tensor and of the temperature, T, in the following form [22–24]

$$\Psi=\Psi(\boldsymbol{C}^e,\boldsymbol{C}^t,T)=\psi^e(\boldsymbol{C}^e)+\psi^t(\boldsymbol{E}^t,T) \tag{10}$$

where $\psi^e(\boldsymbol{C}^e)$ is a hyperelastic strain energy function and $\boldsymbol{E}^t=(\boldsymbol{C}^t-\boldsymbol{1})/2$ is the transformation strain. We remark that in proposing decomposition (10), we have assumed the same material behavior for the austenite and martensite phases. In addition, we assume $\psi^e(\boldsymbol{C}^e)$ to be an isotropic function of $\boldsymbol{C}^e$; it can be therefore expressed as

$$\psi^e(\boldsymbol{C}^e)=\psi^e(I_{\boldsymbol{C}^e},II_{\boldsymbol{C}^e},III_{\boldsymbol{C}^e}) \tag{11}$$

where $I_{\boldsymbol{C}^e},II_{\boldsymbol{C}^e},III_{\boldsymbol{C}^e}$ are the invariants of $\boldsymbol{C}^e$. We also define ψ^t in the following form [8,22,27–29]

$$\psi^t(\boldsymbol{E}^t,T)=\tau_M(T)\|\boldsymbol{E}^t\|+\tfrac{1}{2}h\|\boldsymbol{E}^t\|^2+\mathcal{I}_{\varepsilon_L}(\|\boldsymbol{E}^t\|) \tag{12}$$

where $\tau_M(T)=\beta\langle T-T_0\rangle$ and β, T_0 and h are material parameters; the MacCauley brackets calculate the positive part of the argument, i.e., $\langle x\rangle=(x+|x|)/2$, and the norm operator is defined as $\|\boldsymbol{A}\|=\sqrt{\boldsymbol{A}:\boldsymbol{A}^T}$, with $\boldsymbol{A}:\boldsymbol{B}=A_{ij}B_{ij}$.

Moreover, in Eq. (12) we also use the indicator function $\mathcal{I}_{\varepsilon_L}$ defined as

$$\mathcal{I}_{\varepsilon_L}(\|\boldsymbol{E}^t\|)=\begin{cases}0 & \text{if } \|\boldsymbol{E}^t\|\le\varepsilon_L\\ +\infty & \text{otherwise}\end{cases} \tag{13}$$