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Dynamic analysis of a laminated cylindrical shell with piezoelectric layers under dynamic loads

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ABSTRACT

Dynamic analysis is presented for simply supported laminated cylindrical shell with orthotropic layers bounded with piezoelectric layers, subjected to local ring/pinch loads. The piezoelectric layers serve as sensor/actuator. The governing elasticity equations are reduced to ordinary differential equations by means of trigonometric function expansion. The resulting equations are solved by Galerkin's finite element in radial direction. The static results are compared with similar ones. The convergence is studied and natural frequencies are obtained. The radius to thickness ratio and band load width effect on dynamic behaviour is studied. Time responses for actuated shell are presented for different shell laminations.

1. Introduction

Smart structures equipped with piezoelectric materials have been widely used as distributed sensors and actuators in the area of active structural control. On the other hand, lightweight high-modulus laminated shells of revolution have found extensive applications. Therefore, shell-type smart structures containing piezoelectric layers have been focused. Accurate prediction of their dynamic behaviour demands three-dimensional elasticity based numerical tools. These solutions are needed to assess the accuracy of approximate shell theories. The basic theories for the modeling of piezoelectric materials have been given in many contributions, in particular in the pioneering works of Tiersten [1]. System equations for piezoelectric shell vibrations were derived, using Hamilton's principle and linear piezoelectricity [2]. Many researchers have studied the free vibration of laminated cylindrical shells. Civalek [3] carried out the free vibration analysis of laminated cylindrical shell, using Love's first approximation thin shell theory and discrete singular convolution (DSC) method. Hussein and Heyliger [4] analysed the free vibration of laminated cylindrical shell with piezoelectric layer by means of a discrete layer shell theory and finite element method. The coupled displacement and electrical field equations were derived for a piezoelectric cylindrical shell, based on third order shear deformation theory by PintoCarreia et al. [5], the equations were solved by finite element method. An exact three-dimensional solution for static response of a simply supported laminated piezoelectric cylinder was presented by Heyliger [6]. The exact solution for simply supported multilayered orthotropic cylindrical shell with finite length and piezoelectric layer as sensor and actuator subjected to axisymmetric thermo-electro-mechanical loading was considered by Chen and Shen [7]. Kapuria et al. [8] studied the exact solution for cylindrical piezoelectric shell under various static loads. Chandrashekhara and Nanjunda Rao presented a three-dimensional elasticity solution for an infinite laminated circular cylindrical shell subjected to banded and distributed pinch static loads [9]. Analytical solutions to the radial polarized, piezo-electric thin cylindrical shell based on Kirchhoff's hypo-thesis expressing the axial and radial components of displacement in terms of exponential terms were presented by Ebenezer and Abraham [10]. Shakeri et al. [11] obtained the dynamic response of laminated anisotropic cylindrical panels subjected to dynamic load based on 3D elasticity solution. The 3D-elasticity analysis of laminated cylinder with piezoelectric sensor and actuator layers was presented by Shakeri et al. [12]. A finite element formulation using the layerwise theory, developed for laminated cylindrical shell with piezoelectric layers subjected to dynamic load, besides the 3D elasticity solution has been studied [13]. The elasticity solution for simply supported, laminated cylindrical shell with piezoelectric layer subjected to dynamic load has been presented by the authors [14]. The resulting equations are solved by Galerkin's finite element in radial direction.

Recently a finite element formulation based on the first order shear deformation theory is presented to model the dynamic response of laminated composite shells containing piezoelectric

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Notations

$A, B, \ldots, E'''$ constant coefficients derived in elastic solution
c_{ij} $(i,j=1\ldots6)$ elastic constants
E_{ii} $(i=r,\theta,z)$ elastic modulus
H thickness of the shell
L length of shell
R radius of the shell
u_r, u_θ, u_z radial, circumferential, axial displacement components
$[e]$ piezoelectric coupling constants matrix
$[\eta]$ dielectric constants matrix
D_i $(i=r,\theta,z)$ electric displacement components
E_i $(i=r,\theta,z)$ electric field components
ψ electric potential
P_o maximum applied load
Dx width of the band load
V_{out} applied voltage to the actuator
M, N number of Fourier terms
$\left\{ \{U_r(t)\} \quad \{U_\theta(t)\} \quad \{U_z(t)\} \quad \{\Psi(t)\} \right\}^e$ degree of Freedom vector
$\{N\}$ shape function vector
ϕ_i $(i=r,\theta,z)$ generalized coordinates for displacement
ρ density of shell material
λ_{mn} Eigen values of the shell vibration
ω natural frequency (rad/s)

sensors and actuators subjected to electrical, mechanical and thermal loadings [15]. Negative velocity feedback control algorithm is used to actively control the dynamic response of structure.

In the present work, the elasticity solution of cross-ply laminated cylindrical shell with piezoelectric layer, subjected to dynamic local loading is presented. The cylindrical shell with finite length is simply supported at both ends and elasticity approach is used. The highly coupled partial differential equations are reduced to ordinary differential equations by means of trigonometric function expansion in plane directions. The resulting equations are solved by finite element method. Stress analysis and vibrational behaviour are presented for different shell thicknesses and are compared for different ring loads widths.

2. Problem formulation

The linear constitutive equations for a piezoelectric material are given as follows [2]:

$$\sigma = C\varepsilon - e^T E, \quad D = e\varepsilon + \eta E \tag{1}$$

where the superscript T denotes the transpose of a matrix. The components of stress (σ), strain (ε), electric field (E) and electric displacement vector (D) are given in cylindrical coordinate system (r, θ, z), as follows:

$$\begin{aligned}
&\sigma = [\sigma_r \quad \sigma_\theta \quad \sigma_z \quad \tau_{\theta z} \quad \tau_{rz} \quad \tau_{r\theta}]^T, \quad E = [E_r \quad E_\theta \quad E_z]^T \\
&\varepsilon = [\varepsilon_r \quad \varepsilon_\theta \quad \varepsilon_z \quad \gamma_{\theta z} \quad \gamma_{rz} \quad \gamma_{r\theta}]^T, \quad D = [D_r \quad D_\theta \quad D_z]^T
\end{aligned} \tag{2a}$$

The matrices $[C]$, $[e]$ and $[\eta]$ denote, respectively, the elastic stiffness, piezoelectric and dielectric constants of the orthotropic piezoelectric material layer which are as follows:

$$\begin{aligned}
&[C] = \begin{bmatrix} c_{11} & c_{12} & c_{13} & 0 & 0 & 0 \\ c_{12} & c_{22} & c_{23} & 0 & 0 & 0 \\ c_{13} & c_{23} & c_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{66} \end{bmatrix}, \\
&[e] = \begin{bmatrix} e_{11} & e_{21} & e_{31} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & e_{62} \\ 0 & 0 & 0 & 0 & e_{53} & 0 \end{bmatrix}, \quad [\eta] = \begin{bmatrix} \eta_{11} & 0 & 0 \\ 0 & \eta_{22} & 0 \\ 0 & 0 & \eta_{33} \end{bmatrix}
\end{aligned} \tag{2b}$$

Piezoelectric material with hexagonal symmetric structure exhibits transverse isotropic behaviour relative to its polarization axis. The three-dimensional equations of motion in the absence of body force are

$$\begin{aligned}
&\frac{\partial\sigma_r}{\partial r} + \frac{\sigma_r - \sigma_\theta}{r} + \frac{1}{r}\frac{\partial\tau_{r\theta}}{\partial\theta} + \frac{\partial\tau_{rz}}{\partial z} = \rho\frac{\partial^2 u_r}{\partial t^2} \\
&\frac{1}{r}\frac{\partial\sigma_\theta}{\partial\theta} + \frac{\partial\tau_{\theta z}}{\partial z} + \frac{\partial\tau_{r\theta}}{\partial r} + 2\frac{\tau_{r\theta}}{r} = \rho\frac{\partial^2 u_\theta}{\partial t^2} \\
&\frac{\partial\sigma_z}{\partial z} + \frac{\partial\tau_{rz}}{\partial r} + \frac{1}{r}\frac{\partial\tau_{\theta z}}{\partial\theta} + \frac{1}{r}\tau_{rz} = \rho\frac{\partial^2 u_z}{\partial t^2}
\end{aligned} \tag{3a}$$

The charge equation of electrostatics is given by Tiersten [1]

$$\frac{\partial D_r}{\partial r} + \frac{D_r}{r} + \frac{\partial D_\theta}{r\partial\theta} + \frac{\partial D_z}{\partial z} = 0 \tag{3b}$$

The strain–displacement and the electric field–electric potential relations of the piezoelectric medium are written as

$$\begin{aligned}
&\varepsilon_r = \frac{\partial u_r}{\partial r} \quad \gamma_{\theta z} = \frac{\partial u_\theta}{\partial z} + \frac{1}{r}\frac{\partial u_z}{\partial\theta} \quad E_r = -\frac{\partial\psi}{\partial r} \\
&\varepsilon_\theta = \frac{1}{r}\left(u_r + \frac{\partial u_\theta}{\partial\theta}\right) \quad \gamma_{r\theta} = \frac{1}{r}\left(\frac{\partial u_r}{\partial\theta} - u_\theta + r\frac{\partial u_\theta}{\partial r}\right) \quad E_\theta = -\frac{1}{r}\frac{\partial\psi}{\partial\theta} \\
&\varepsilon_z = \frac{\partial u_z}{\partial z} \quad \gamma_{rz} = \frac{\partial u_z}{\partial r} + \frac{\partial u_r}{\partial z} \quad E_z = -\frac{\partial\psi}{\partial z}
\end{aligned} \tag{4}$$

By combining Eq. (4) with Eq. (1), the stress and electrical displacement component will be obtained as follows:

$$\begin{aligned}
&\sigma_r = c_{11}\frac{\partial u_r}{\partial r} + c_{12}\left(\frac{u_r}{r} + \frac{1}{r}\frac{\partial u_\theta}{\partial\theta}\right) + c_{13}\frac{\partial u_z}{\partial z} + e_{11}\frac{\partial\Psi}{\partial r} \\
&\sigma_\theta = c_{12}\frac{\partial u_r}{\partial r} + c_{22}\left(\frac{u_r}{r} + \frac{1}{r}\frac{\partial u_\theta}{\partial\theta}\right) + c_{23}\frac{\partial u_z}{\partial z} + e_{21}\frac{\partial\Psi}{\partial r} \\
&\sigma_z = c_{13}\frac{\partial u_r}{\partial r} + c_{23}\left(\frac{u_r}{r} + \frac{1}{r}\frac{\partial u_\theta}{\partial\theta}\right) + c_{33}\frac{\partial u_z}{\partial z} + e_{31}\frac{\partial\Psi}{\partial r} \\
&\tau_{rz} = c_{55}\left(\frac{\partial u_z}{\partial r} + \frac{\partial u_r}{\partial z}\right) + e_{53}\left(\frac{\partial\Psi}{\partial z}\right) \\
&\tau_{\theta z} = c_{44}\left(\frac{\partial u_\theta}{\partial z} + \frac{1}{r}\frac{\partial u_z}{\partial\theta}\right) \\
&\tau_{r\theta} = c_{66}\left(\frac{\partial u_\theta}{\partial r} + \frac{1}{r}\left(\frac{\partial u_r}{\partial\theta} - u_\theta\right)\right) + e_{62}\left(\frac{1}{r}\frac{\partial\Psi}{\partial\theta}\right) \\
&D_r = e_{11}\frac{\partial u_r}{\partial r} + e_{21}\left(\frac{u_r}{r} + \frac{1}{r}\frac{\partial u_\theta}{\partial\theta}\right) + e_{31}\frac{\partial u_z}{\partial z} - \eta_{11}\frac{\partial\Psi}{\partial r} \\
&D_\theta = e_{62}\left(\frac{\partial u_\theta}{\partial r} + \frac{1}{r}\left(\frac{\partial u_r}{\partial\theta} - u_\theta\right)\right) - \eta_{22}\left(\frac{1}{r}\frac{\partial\Psi}{\partial\theta}\right) \\
&D_z = e_{53}\left(\frac{\partial u_z}{\partial r} + \frac{\partial u_r}{\partial z}\right) - \eta_{33}\frac{\partial\Psi}{\partial z}
\end{aligned} \tag{5}$$

The other components are derived in the same way. After substituting these components into Eq. (3) and factorizing the similar u's, the governing equations of equilibrium in terms of displacement and electric potential for each layer of cylindrical