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Eigenfrequencies of symmetric planar frames with semi-rigid joints using weighted graphs

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Abstract

The main aim of this paper is to extend the recently developed methods for calculating the free vibration analysis of planar symmetric frames to include the effect of semi-rigidity of the joints. This is achieved by decomposing a symmetric weighted graph model into two submodels and using canonical forms in such a manner that the union of the eigenvalues of the submodels result in the eigenvalues of the entire model. Thus the eigenfrequencies of the frame is obtained in an efficient manner. Here, only the free vibration of frames with linear behavior is studied.

Keywords: Symmetry; Eigenfrequencies; Semi-rigid frame; Decomposition; Weighted graph

1. Introduction

Symmetry has been widely used in science and engineering, Hargittai [1], Gruber [2], Glockner [3], Zingoni [4], Zingoni et al. [5]. Many eigenvalue problems arise in many scientific and engineering problems, Livesley [6], Jennings and McKeown [7], Bathe and Wilson [8]. While the basic mathematical ideas are independent of the size of matrices, the numerical determination of eigenvalues and eigenvectors becomes more complicated as the dimensions of matrices increase. Special methods are beneficial for efficient solution of such problems, especially when their corresponding matrices are highly sparse.

Methods are developed for decomposing and healing the graph models of structures, in order to calculate the eigenvalues of matrices and graph matrices with special patterns. The eigenvectors corresponding to such patterns for the symmetry of Form I, Form II and Form III are studied in references, Kaveh and Sayarinejad [9,10], and the applications to vibrating mass-spring systems and frame structures are developed in Kaveh and Sayarinejad [11] and Kaveh and Salimbahrami [12], respectively. These forms are also applied to calculating the buckling load of symmetric mechanical systems [13,14].

The main aim of this paper is to extend the method developed in Kaveh and Dadfar [15] for calculating the eigenfrequencies of the frames with rigid joints to include the effect of semi-rigidity of the joints. This is achieved by decomposing a symmetric model into two submodels and then performing healing in such a manner that the union of the eigenvalues of the healed submodels result in the eigenvalues of the entire model. Thus the eigenfrequencies of the frame is obtained in an efficient manner.

In this paper, only the free vibration of planar frames with linear behavior is considered, however, the method can be extended to space frames and forced vibration of frame structures.

2. Transformation of matrices to canonical forms

In this section, an $N \times N$ symmetric matrix [**M**] is considered with all entries being real. For three special canonical forms, the eigenvalues of [**M**] are obtained using the properties of its submatrices.

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2.1. *Canonical Form I*

In this case [**M**] has the following pattern:

$$[\mathbf{M}]=\left[\begin{array}{c|c}[\mathbf{A}]_{n\times n} & [\mathbf{0}]_{n\times n}\\ \hline [\mathbf{0}]_{n\times n} & [\mathbf{A}]_{n\times n}\end{array}\right]_{N\times N}, \tag{1}$$

with $N=2n$.

Considering the set of eigenvalues of the submatrix [**A**] as $\{\lambda(\mathbf{A})\}$, the set of eigenvalues of [**M**] can be obtained as

$$\{\lambda(\mathbf{M})\}=\{\lambda(\mathbf{A})\}\overline{\cup}\{\lambda(\mathbf{A})\}. \tag{2}$$

Since $\det(\mathbf{M})=\det(\mathbf{A})\times\det(\mathbf{A})$, the above relation becomes obvious. The sign $\overline{\cup}$ simply indicates the collection of the eigenvalues of the submatrices.

2.2. *Canonical Form II*

For this case, matrix [**M**] can be decomposed into the following form:

$$[\mathbf{M}]=\left[\begin{array}{c|c}[\mathbf{A}]_{n\times n} & [\mathbf{B}]_{n\times n}\\ \hline [\mathbf{B}]_{n\times n} & [\mathbf{A}]_{n\times n}\end{array}\right]_{N\times N}. \tag{3}$$

The eigenvalues of [**M**] can be calculated as

$$\{\lambda(\mathbf{M})\}=\{\lambda(\mathbf{C})\}\overline{\cup}\{\lambda(\mathbf{D})\}, \tag{4}$$

where

$$[\mathbf{C}]=[\mathbf{A}]+[\mathbf{B}] \quad \text{and} \quad [\mathbf{D}]=[\mathbf{A}]-[\mathbf{B}]. \tag{5}$$

[**C**] and [**D**] are called *condensed submatrices* of [**M**]. The proof of this form can be considered as the special case of the proof for Form III, and it is not repeated for brevity.

2.3. *Canonical Form III*

This form has a Form II submatrix augmented by some rows and columns as shown in the following:

$$[\mathbf{M}]=\left[\begin{array}{cc|cccccc}
 & & & & L_{11} & \cdots & L_{1k}\\
 [\mathbf{A}] & & [\mathbf{B}] & & L_{21} & \cdots & L_{2k}\\
 & & & & & \cdots & \\
\hline
 & & & & L_{n1} & & L_{nk}\\
 & & & & L_{11} & \cdots & L_{1k}\\
 [\mathbf{B}] & & [\mathbf{A}] & & L_{21} & & L_{2k}\\
 & & & & L_{n1} & \cdots & L_{nk}\\
C(2n+1,1) & . & C(2n+1,2n) & & C(2n+1,2n+1) & \cdots & C(2n+1,2n+k)\\
. & . & . & & . & \cdots & .\\
Z(2n+k,1) & . & Z(2n+k,2n) & & Z(2n+k,2n+1) & \cdots & Z(2n+k,2n+k)
\end{array}\right]. \tag{6}$$

where [**M**] is a $(2n+k)\times(2n+k)$ matrix, with a $2n\times 2n$ submatrix with the pattern of Form II, and k augmented columns and rows. The entries of the augmented columns are repeated in the first and second block for each column, and all the entries of [**M**] are real numbers.

Now [**D**] is obtained as $[\mathbf{D}]=[\mathbf{A}]-[\mathbf{B}]$, and [**E**] is constructed as the following:

$$[\mathbf{E}]=\left[\begin{array}{cc|ccc}
 & & L_{11} & \cdots & L_{1k}\\
 [\mathbf{A}+\mathbf{B}] & & L_{21} & \cdots & L_{2k}\\
 & & . & \cdots & .\\
 & & L_{n1} & \cdots & L_{nk}\\
\hline
C(2n+1,1)+C(2n+1,n+1) & . & C(2n+1,2n+1) & \cdots & C(2n+1,2n+k)\\
. \quad . & . & . & \cdots & .\\
Z(2n+k,1)+Z(2n+k,n+1) & . & Z(2n+k,2n+1) & \cdots & Z(2n+k,2n+k)
\end{array}\right]. \tag{7}$$