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Equivalent axial stiffness of various components in bolted joints subjected to axial loading

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Abstract

Managing the axial stiffness of various components in a bolted joint is a major industrial concern for modelling different tightening processes and for accurate fatigue dimensioning. This paper presents a new approach for calculating the axial stiffness of the several elements of a bolt (the head and the engaged part), the nut and the fastened plates. Finite element modelling based on deformation energy improves the existing models and take in to account the type of materials and coefficients of friction of various elements in contact. From these corrections, approaches for axial stiffness calculation based on empirical formulas are proposed for easier application and for future FE modelling bolted joints using beam elements. Finally, the theoretical study is validated by an original experimental approach.

Keywords: Bolted joint; Angle-controlled tightening; Axial stiffness; Bolt; Nut; Tapping; Contact; Modelling; Finite elements

1. Introduction

Stiffness of the subassemblies in a bolted joint reflects the overall behaviour of the assembly subjected axial loading which is mainly displacement along the axis of the bolts. Thus, managing the equivalent stiffness of the bolt, the nut and subassemblies is necessary for dimensioning in fatigue, modelling the tightening process and investigating the behaviour of these assemblies under thermal stresses.

The 1986 VDI2230 recommendation which is the most frequently used in industrial calculations and specialised research works like those developed by Bickford and Nassar [1], Guillot [2] was always questionable [3] and even completely modified in some references like the 2003 VDI2230 [4]. A benchmark of developed approaches anterior to 1990 can be found in Lenhoff et al. [5]. However, it was the development of finite elements which led up to more accurate models. One should note that the main objective remains displacement along the axis of the bolt. In this framework, Wileman et al. [6] considered the washer as a rigid body in simulating the interaction between the head of the bolt and the part. Unfortunately, this approach appeared to be irrealistic and inaccurate as shown by Guillot [3], Massol [7] and Zadoks [8]. Lehnhoff et al. [5,9,10] calculated an average equivalent displacement for the nodes at the contact zone to simulate this interaction.

To validate this approach, Massol [7] conducted experimental measurements. Unfortunately, it was difficult to accurately determine the deformation of the subassemblies along the assembly axis. Thus, the results seemed mediocre.

In this paper, a new approach for stiffness calculation is presented: this approach takes into account the various geometrical parameters, the coefficient of friction and the materials properties in the case of axisymmetric loading.

Three-dimensional finite elements (FEM) are conducted to calculate the apparent stiffness of the subassemblies using the deformation energy method. Then, results are compared to previous studies and validated by an original experimental approach.

Finally, the various ratios necessary for stiffness calculation are integrated in an empirical formula that can be easily programmable in a tool for industrial design office use.

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Nomenclature

A_{d3}	thread root area (mm^2)
A_p	equivalent cross-section of fastened parts (mm^2)
A_p^*	dimensionless equivalent cross-section of fastened parts
A_s	bolt stress area (mm^2)
A_0	bolt nominal cross-section (mm^2)
D	nut nominal diameter (mm)
d	bolt nominal diameter (mm)
D_a	diameter under bolt head (mm)
D_ext	nut external diameter (mm)
D_p	fastened plates diameter (mm)
D_p^*	fastened plates dimensionless diameter
D_t	bolt hole diameter (mm)
D_t^*	bolt hole dimensionless diameter
E_bolt	bolt modulus of elasticity ($\mathrm{N/mm}^2$)
E_head	bolt head modulus of elasticity ($\mathrm{N/mm}^2$)
E_nut	nut modulus of elasticity ($\mathrm{N/mm}^2$)
E_part	fastened plates modulus of elasticity ($\mathrm{N/mm}^2$)
E_tapping	tapped part modulus of elasticity ($\mathrm{N/mm}^2$)
F_tot	applied axial load (N)
H	nut standardised height (mm)
h	height of bolt head (mm)
$K_\mathrm{bolt\text{–}nut}$	stiffness of the nut and engaged part of the bolt (N/mm)
K_head	bolt head stiffness (N/mm)
K_p	stiffness of fastened parts (N/mm)
K_p^*	dimensionless stiffness of fastened parts
K_tapping	stiffness of a tapped hole (N/mm)
$K_\mathrm{bolt\text{–}tapping}$	tapped hole and thread stiffness (N/mm)
L_p	height of bolted parts (mm)
L_p^*	dimensionless height of bolted parts
L_0	length of bolt cylindrical part (mm)
L_1	length of bolt threaded part (mm)
p	pitch of thread (mm)
$W_\mathrm{e\ nut}$	deformation energy induced in the nut (N mm)
$W_\mathrm{e\ part}$	deformation energy induced in the bolted parts (N mm)
α_bolt	correcting factor of the engaged part of the bolt
$\alpha_\mathrm{bolt\text{–}nut}$	stiffness correcting factor of the nut and the engaged part of the bolt
$\alpha_\mathrm{bolt\text{–}tapping}$	stiffness correcting factor of a tapped hole and the engaged part of the bolt
α_head	stiffness correcting factor of the bolt head
α_threaded	stiffness correcting factor of the bolt threaded part
α_tapping	stiffness correcting factor of a tapped hole
δ_0	displacement due to clamping (on preloading F_0) (mm)
μ	coefficient of friction
μ_eq	equivalent coefficient of friction
μ_1	coefficient of friction between the bolt and the nut
μ_2	coefficient of friction between the nut and the fastened plates
θ	tightening angle (degrees)

2. Problem set up

2.1. General approach

In the aim of developing a new general calculation method, the overall assembly is partitioned into functional parts: the threaded portion of the bolt, the bolt head, the threads engaged in the nut, the fastened plates and the threads engaged in the tapped subassembly (see Fig. 1). The local approach consists of calculating the local rigidity of each of these functional parts. It is then extended to a more global approach to calculate the global stiffness of the bolted assembly. This classical approach is similar to the one used in VDI 2230 recommendation [11].

2.2. Calculation of the equivalent stiffness

Each subassembly is assimilated to a spring with an equivalent stiffness (see Fig. 3). From finite element analysis, one can conclude (see Fig. 3a) that under loading, the displacement of the upper part of the screw (δ) is proportional to the applied load F [12]. Calculating the stiffness K_B and K_p of each element apart is justified since due to local deformations at the interface head-part, it is difficult to accurately estimate the displacement along the axis, and consequently the length variation (see Fig. 2).

Using the elastic deformation energy for each element, the equivalent stiffness can be deduced. Applying the principle of energy conservation to the considered system (see Fig. 2a), and considering a free unfrictional contact in the rigid plane (x, z):

$$\tfrac{1}{2}F\delta = W_\mathrm{B} + W_\mathrm{p} + W_\mathrm{f}, \qquad (1)$$

where W_B is the elastic deformation energy of the bolt, W_p the elastic deformation energy of the part, W_f the dissipated frictional energy at the interface head-part, and $\frac{1}{2}F\delta$ the work of the external forces.

Consequently, the two cases which can be investigated are presented below.

2.2.1. Unfrictional contact under head

$$\tfrac{1}{2}F\delta = W_\mathrm{B} + W_\mathrm{p}. \qquad (2)$$

Since the finite elements model show a linear behaviour, the stiffness of the springs can be calculated using the following relations:

$$K_\mathrm{B} = \frac{F^2}{2W_\mathrm{B}} \quad \text{and} \quad K_\mathrm{p} = \frac{F^2}{2W_\mathrm{p}}. \qquad (3)$$