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A geometrically nonlinear finite element formulation for shells using a particular linearization method

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Abstract

In this paper a particular linearization method is used to derive the updated Lagrangian finite element formulation for geometrically nonlinear analysis of shell structures. Derivation of the formulation is based on rewriting the Green–Lagrange strain and the second Piola–Kirchhoff stress as two second-order functions in terms of a through-the-thickness parameter. Substitution of these two functions into the equilibrium equation, from principle of virtual work, and then using a Taylor series expansion for the nonlinear term lead to a modified linearized incremental equation. In this procedure the stiffness matrices and the internal force vector are consistently derived following the linearization. The results are compared with those of other researchers and good agreements are observed. Advantages of the proposed approach are assessed and comparisons with available solutions are presented.

Keywords: Geometrically nonlinear; Linearization; Finite element method; Shell structure

1. Introduction

In many plates and shells structures, for optimal design, large deformation analysis is unavoidable. In the last decades, the development of efficient computational models for the nonlinear analysis of shells has been one of the most important research activities [1]. This is partially motivated by the need to analyze new materials such as composite and functionally graded shells [1–7].

Bathe [8] has presented a detailed discussion on Lagrangian formulation for large deformation analysis of structures. Also the proposed degenerated shell element by Ahmad et al. [9] has always been found to be attractive in solving shell problems.

Based on Bathe's standard Lagrangian finite element formulation for degenerated shell element, many have been done by researchers who have struggled to suggest new, accurate and optimal strategies for geometrically nonlinear analysis of shell structures [10–12]. Meanwhile, other researchers have been evaluating other techniques [13–18].

In large deformations due to continuous change in configuration, an incremental procedure is needed for solving the equations of equilibrium, which are expressed by the principle of virtual work in a current configuration of the structure. While the current configuration is unknown, it is necessary to write the principle of virtual work for the body at a known equilibrium configuration. In this case, a strongly nonlinear equation will be obtained which should be linearized. According to this statement, some ideas on linearization techniques formed and have been proposed by researchers in the literature [8,19,20].

During the linearization process a significant loss occurs in accuracy of the Lagrangian formulation. This paper is motivated to propose a further simplification during the linearization process to avoid this inaccuracy.

In this paper, a finite element formulation is presented for geometrically nonlinear analysis of shell structures. This formulation is based on a modified linearization approach which is implemented on the Bathe's standard updated Lagrangian

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(UL) formulation after making an appropriate decomposition in second Piola–Kirchhoff (PK2) stress and Green–Lagrange (GL) strain tensors. In order to show the accuracy and capability of the presented finite element formulation, some examples are solved and the results are compared with those available in the literature.

2. Kinematics of shell element

The eight-noded degenerated shell element, Fig. 1, is a continuum based element that has been introduced in finite element books, for example [8,21,22]. This degenerate shell element has been used for modeling large rigid body motion, large displacement with small rotations. In a case problem with large rotation, due to some singularity in rotational degrees of freedom the element fails to model finite deformation [1]. The configuration of this shell element having thickness a is expressed by

$$^{t}x_{i(\xi,\eta,\zeta)} = \sum_{k=1}^{8} h_{k(\xi,\eta)}\,{}^{t}x_i^k + \frac{\zeta}{2}\sum_{k=1}^{8} h_{k(\xi,\eta)} a^k\,{}^{t}V_{ni}^k, \quad i = 1, 2, 3, \tag{1}$$

where h_k is the shape function corresponding to nodal point k. Also, ${}^{t}V_{ni}^k$ is the local normal vector component i at node k at time t.

According to geometry interpolation relation (1) and Fig. 1, in a three-dimensional space, two vectors are required to define the geometry of the shell element. One vector expresses the configuration of the shell middle surface using two curvilinear coordinates ξ and η. The next vector using unit normal vector ($\mathbf{V}_n^k$) expresses any position between the top and the bottom surfaces of the shell in the ζ (thickness) direction.

Using Eq. (1), the displacement field of the isoparametric shell element at time t is obtained as

$$^{t}u_{i(\zeta,\eta,\xi)} = \sum_{k=1}^{8} h_{k(\xi,\eta)}\,{}^{t}u_i^k + \frac{\zeta}{2}\sum_{k=1}^{8} h_{k(\xi,\eta)} a^k (-{}^{t}V_{2i}^k\,{}^{t}\alpha^k + {}^{t}V_{1i}^k\,{}^{t}\beta^k). \tag{2}$$

In Eq. (2) the orthogonal vectors $\mathbf{V}_1^k$ and $\mathbf{V}_2^k$ are used to express the normal vector components $\mathbf{V}_n^k$ in terms of rotations α^k and β^k, where α^k and β^k are incremental rotation angles of the normal vector $\mathbf{V}_n^k$ about the vectors $\mathbf{V}_1^k$ and $\mathbf{V}_2^k$ from the configuration at time t to the configuration at time $t + \Delta t$.

3. Nonlinear governing equation

The equilibrium equation of a deforming body at time $\tau = t + \Delta t$ based on UL incremental analysis approach using the principle of virtual displacements is expressed as follows:

$$\int_{^{t}V} {}^{t+\Delta t}_{t}S_{ij}\,\delta^{t+\Delta t}{}_{t}E_{ij}\,\mathrm{d}^{t}V = {}^{t+\Delta t}\Re, \tag{3}$$

where ${}^{t+\Delta t}_{t}S_{ij}$ and ${}^{t+\Delta t}_{t}E_{ij}$ are the PK2 stress and the GL strain tensors, respectively. These two tensors correspond to increment $\tau = t + \Delta t$ but are measured using configuration at $\tau = t$. The parameter ${}^{t+\Delta t}\Re$ represents the external virtual work. The PK2 stress and GL strain tensors are an energy conjugate pair [23] and both of them are not affected by rigid body rotations. The incremental form of the GL strain tensor is as follows:

$${}^{t+\Delta t}_{t}E_{ij} = \tfrac{1}{2}\left({}^{t+\Delta t}_{t}u_{i,j} + {}^{t+\Delta t}_{t}u_{j,i} + {}^{t+\Delta t}_{t}u_{k,i}\,{}^{t+\Delta t}_{t}u_{k,j}\right). \tag{4}$$

Also, the relation between the PK2 stress components S_{ij} and the Cauchy stress components σ_{mn} is written as follows:

$${}^{t+\Delta t}\sigma_{mn} = \det\left({}^{t+\Delta t}_{t}x_{m,i}\right){}^{t+\Delta t}_{t}x_{m,i}\,{}^{t+\Delta t}_{t}S_{ij}\,{}^{t+\Delta t}_{t}x_{n,j}, \tag{5}$$

where ${}^{t+\Delta t}_{t}x_{m,i} = \partial^{t+\Delta t}x_m/\partial^{t}x_i$ represents the deformation gradient of the increment.

4. Linearization of the nonlinear equation

Substitution of displacement field (2) into Eq. (4) yields a second-order function for expressing the GL strain tensor as follows:

$${}^{t+\Delta t}_{t}E_{ij} = {}^{t+\Delta t}_{t}E_{ij}^{(0)} + \zeta\,{}^{t+\Delta t}_{t}E_{ij}^{(1)} + \zeta^2\,{}^{t+\Delta t}_{t}E_{ij}^{(2)}, \tag{6}$$

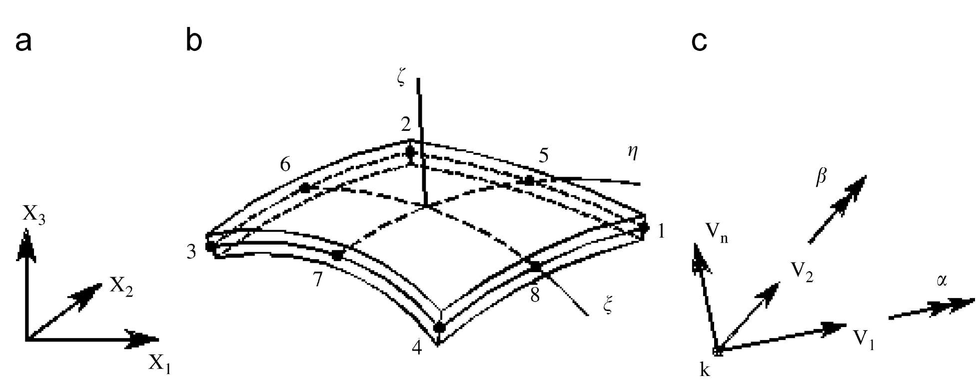

Fig. 1. (a) Global coordinate system, (b) the 8-noded degenerated shell element with natural coordinate system, (c) unit normal vectors on node k and two rotation local degrees of freedom.