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Dynamic analysis of fluid–structure interaction problems with modal methods using pressure-based fluid finite elements

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Abstract

The present paper deals with numerical developments performed in the finite element code ANSYS in order to produce coupled fluid–structure dynamic analysis with pressure-based formulation, using modal and spectral methods. Enhancement of the modelling possibilities within the ANSYS code is carried out with implementation of fluid–structure symmetric formulations for elasto-acoustic and hydro-elastic problems, using the so-called symmetric ($\mathbf{u}$, p, φ) and ($\mathbf{u}$, η, φ) formulations. Using symmetric formulation enables linear dynamic analysis with modal projection techniques for a fluid–structure coupled system. The paper briefly recalls the basic principles of such methods in the context of FSI. Validation of the developments performed in the ANSYS code is exposed, focusing in particular on the calculation of effective mass for coupled eigenmodes. Industrial application is also presented and gives a validation test case for modal and spectral methods with FSI modelling.

Keywords: Fluid–structure interaction; Fluid; Spectral analysis; Symmetric formulations; Effective modal mass; Finite element code validation

1. Introduction

Dynamic response of linear systems subjected to dynamic loading, such as shock or seism, with finite element procedures is of paramount importance in many engineering applications [1]. Taking fluid–structure interaction into account in such problems is made possible by the development of finite element or boundary element methods [2].

Although such methods have been firmly validated from the theoretical, numerical and even experimental points of view [3,4], their application for design purposes is still not possible with some industrial finite element codes.

In a previous paper [5], numerical enhancement of the finite element code ANSYS, of wide use in academia and industry, for finite element modelling of coupled fluid–structure systems has been exposed and validated. It has been highlighted that the use of a newly implemented symmetric coupled formulation instead of the non-symmetric coupled formulation currently available in the code made it possible to perform coupled modal analysis on complex industrial problems with reasonable computational time.

The present paper is devoted to validation of the use of these symmetric formulations for the dynamic analysis of linear coupled fluid–structure system, with modal methods. In Section 2, a brief overview of various dynamic analysis methods is recalled. In particular, the basic principles of spectral methods, of wide use in seismic engineering [6], are exposed in the context of fluid–structure system. Definition and properties of participation factors and effective masses for coupled eigenmodes are exposed and demonstrated. In Section 3, a brief outline of the development of symmetric coupled formulations in the ANSYS code is exposed. In Section 4, elementary test cases are defined and validation is mainly concerned with the calculation of eigenfrequencies and effective masses. Finally, an industrial application is proposed in Section 5, to conclude validations of the ANSYS code.

The paper also aims at providing users of the ANSYS code—as well as other finite element codes—with test cases to refer to, for the application of modal and spectral analysis with FSI modelling. Applications of the methods exposed in

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the present paper cover a wide range of engineering problems in nuclear, marine, aeronautic and automotive industries, for instance for seismic and vibro-acoustic analysis.

2. Modal methods for dynamic linear fluid–structure problems

2.1. Dynamic analysis of a linear system with time integration methods

The problem of interest in the present paper is to compute the response of a linear system subjected to an imposed acceleration, such as a seism or a shock. The dynamic load on the system is described by the acceleration profile $\gamma(t)$ in a given direction $\mathbf{D}$ and the system response is defined by the evolution of its degrees of freedom $\mathbf{X}(t)$ (e.g. in the context of fluid–structure interaction problems, structure displacement field and fluid pressure and displacement potential fields) in the moving frame. $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ denoting, respectively, the system mass, damping and stiffness matrices, the system dynamic is described by the following equation [7]:

$$\mathbf{M}\ddot{\mathbf{X}}(t) + \mathbf{C}\dot{\mathbf{X}}(t) + \mathbf{K}\mathbf{X}(t) = -\mathbf{M}\mathbf{D}\gamma(t). \tag{1}$$

2.1.1. Direct time integration methods

Computation of the system response can be performed with direct time integration methods, i.e. solving Eq. (1) with finite difference schemes, either with explicit [8] or implicit [9]break approaches. As an example, an implicit finite difference scheme is produced as follows. Using Taylor developments of displacement $\mathbf{X}(t+\delta t)$ and velocity $\dot{\mathbf{X}}(t+\delta t)$ leads to the finite difference approximation of the system acceleration $\ddot{\mathbf{X}}_{k+1} = \ddot{\mathbf{X}}(t_{k+1})$ and velocity $\dot{\mathbf{X}}_{k+1} = \dot{\mathbf{X}}(t_{k+1})$ at time step t_{k+1} using the system acceleration $\ddot{\mathbf{X}}_k = \ddot{\mathbf{X}}(t_k)$, velocity $\dot{\mathbf{X}}_{k+1} = \dot{\mathbf{X}}(t_{k+1})$ and displacement $\mathbf{X}_k = \mathbf{X}(t_k)$ at the previous time step t_k, according to:

$$\ddot{\mathbf{X}}_{k+1} = \frac{\mathbf{X}_{k+1} - \mathbf{X}_k}{\beta\delta t^2} - \frac{\dot{\mathbf{X}}_k}{\beta\delta t} - \left(\frac{1}{2\beta} - 1\right)\ddot{\mathbf{X}}_k + O(\delta t) \tag{2}$$

and

$$\dot{\mathbf{X}}_{k+1} = \dot{\mathbf{X}}_k + \delta t(\alpha\mathbf{X}_{k+1} + (1-\alpha)\mathbf{X}_k) + O(\delta t) \tag{3}$$

with β and α the parameters of the algorithm.

Substituting Eqs. (2) and (3) into Eq. (1), and neglecting terms in $O(\delta t)$ yields:

$$\begin{aligned}\left(\frac{\mathbf{M}}{\beta\delta t^2} + \frac{\mathbf{C}}{\beta\delta t} + \mathbf{K}\right)\mathbf{X}_{k+1} = & -\mathbf{M}\mathbf{D}\gamma_{k+1} + \left(\frac{\mathbf{M}}{\beta\delta t^2} + \frac{\mathbf{C}}{\beta\delta t}\right)\mathbf{X}_k \\ & + \left(\frac{\mathbf{M}}{\beta\delta t} - \mathbf{C}\left(1 - \frac{\alpha}{\beta}\right)\right)\dot{\mathbf{X}}_k \\ & + (\mathbf{M} + \mathbf{C}\alpha\delta t)\left(\frac{1}{2\beta} - 1\right)\ddot{\mathbf{X}}_k.\end{aligned} \tag{4}$$

All terms in the right hand side of Eq. (4) are known quantities; computation of $\mathbf{X}_{k+1}$ is straightforward and requires the inversion of the matrix $\tilde{\mathbf{K}} = \mathbf{M}/\beta\delta t^2 + (\mathbf{C}/\beta\delta t) + \mathbf{K}$. For linear problems, $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ are time independent, thus inversion of $\tilde{\mathbf{K}}$ has to be performed just once at the first time step of the algorithm. This implicit method is the well-known Newmark [10] scheme, which is implemented in the ANSYS code [11].

2.1.2. Modal time integration methods

Direct integration methods are rather time consuming, even with computers nowadays and for rather simple finite element models. It is then more efficient to use a projection of Eq. (1) onto a suitable vector basis in order to solve a set of ordinary differential equations [12]. Projection is made possible by expanding the unknown problem $\mathbf{X}(\mathbf{x}, t)$ as

$$\mathbf{X}(\mathbf{x}, t) = \sum_{n>0} \xi_n(t)\mathbf{X}_n(\mathbf{x}), \tag{5}$$

where $(\mathbf{X}_n)_{n>0}$ is a vector basis and $(\xi_n)_{n>0}$ are the generalised coordinates of vector $\mathbf{X}$. The major interest of decomposition (5) is to separate the time and space dependency of the system degrees of freedom into the coordinates $\xi_n(t)$ on the one hand, and the basis vector $\mathbf{X}_n(\mathbf{x})$ on the other hand.

The most appropriate vector basis is that of the system eigenvectors, those latter being the solutions of the eigenvalue problem:

$$(-\omega_n^2\mathbf{M} + \mathbf{K})\mathbf{X}_n = \mathbf{0} \tag{6}$$

with ω_n the eigenpulsation associated with eigenvector $\mathbf{X}_n$.

When $\mathbf{M}$ and $\mathbf{K}$ are *symmetric* and *positive definite* matrices, the eigenvectors are a basis of the problem unknown vector space and comply with the following orthogonality conditions:

$$\mathbf{X}_n^{\mathrm{T}}\mathbf{M}\mathbf{X}_{n'} = \delta_{n,n'}m_n, \quad \mathbf{X}_n^{\mathrm{T}}\mathbf{K}\mathbf{X}_{n'} = \delta_{n,n'}k_n, \tag{7}$$

where $\delta_{n,n'}$ stands for the Kronecker symbol. m_n and k_n are referred to as the *modal mass* and *modal stiffness* of eigenvector $\mathbf{X}_n$, respectively. The system dynamic behaviour can then be viewed as the superposition of elementary mass-spring systems with mass m_n and spring stiffness k_n, each system oscillating at frequency f_n, given by

$$f_n = \frac{1}{2\pi}\sqrt{\frac{k_n}{m_n}}. \tag{8}$$

Substituting the modal decomposition (5) into Eq. (1), multiplying each term by $\mathbf{X}_n^{\mathrm{T}}$ and using the orthogonality conditions given by Eq. (7) yields the following set of equations:

$$\ddot{\xi}_n(t) + \frac{(\mathbf{X}_n^{\mathrm{T}}\mathbf{C}\mathbf{X}_n)}{m_n}\dot{\xi}_n(t) + \omega_n^2\xi_n(t) = -\kappa_n\gamma(t), \tag{9}$$

where κ_n is the *participation factor* of eigenmode $\mathbf{X}_n$, defined as

$$\kappa_n = \frac{\mathbf{X}_n^{\mathrm{T}}\mathbf{M}\mathbf{D}}{\mathbf{X}_n^{\mathrm{T}}\mathbf{M}\mathbf{X}_n}. \tag{10}$$

The participation factor can be interpreted as a shape factor which indicates how eigenmode $\mathbf{X}_n$ is to respond to the imposed acceleration in direction $\mathbf{D}$: the higher κ_n, the greater