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Propagation of in-plane elastic waves in a composite panel

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Abstract

The paper presents certain results of the analysis of in-plane elastic wave propagation in a composite panel. This problem is solved by the use of the Spectral Element Method. In the current approach the composite panel is modelled by a 36-node spectral membrane finite elements with nodes defined at Gauss–Lobatto–Legendre points. As approximation polynomials the fifth order orthogonal Lagrange polynomials have been used. In order to calculate the element characteristic stiffness and mass matrices the Gauss–Lobatto quadrature has been applied. Numerical calculations are carried out for various orientations of reinforcing fibres within the panel as well as for various volume fractions of the fibres. It is shown that propagation of in-plane elastic waves in composite materials is a more complex phenomenon than in the case of isotropic materials. The velocities of the elastic waves and also the direction of propagation are functions of the fibre orientation and the relative volume fraction of the fibres.

Keywords: Wave propagation; Composite panel; Spectral Finite Element Method

1. Introduction

The use of composite materials in the fields of mechanical and civil engineering has substantially increased over past decades. High performance, strength, stiffness and low weight are the attractive factors which have increased the use of these materials in aerospace, automobile, marine and rail industries [1–6].

Propagation of elastic waves in composite materials has been studied over a considerable period of time. Even though mathematical backgrounds of this phenomenon are well established and developed, wave propagation in real scale engineering structures still remains an open area of research. The main problems in the analysis of propagation of high-velocity elastic waves in distributed composite structures are related to spatial discretisation which must be fine and accurate in order to capture the amplified effect of wave scattering at structural discontinuities. A conventional modal method, when extended to a high-frequency regime, becomes computationallyinefficient since many higher modes that should participate in motion will be misrepresented. For a specific geometry and finite periodic or semi-infinite boundary conditions many different solution techniques have been proposed and reported so far—an excellent overview of these techniques is given in [7]. Previous studies were based mainly on the Finite Difference Method (FDM) [8] and the Finite Element Method (FEM) [9–14]. Also the Boundary Element Method (BEM) [15,16] which utilises surface integrals based on the solutions in terms of special Green's function was applied for modelling and analysis of elastic wave propagation phenomena. In [17,18] the Finite Strip Element Method (FSEM) was applied for the same kind of problems. The main advantage of the latter is that it requires much less storage space for necessary data due to lower level of discretisation and polynomial approximation. An extensive summary of all the above-mentioned techniques used for modelling of ultrasonic waves is given in [19].

A different approach has been proposed in [20,21]. In the Mass–Spring–Lattice Model (MSLM) inertia and stiffness

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properties are calculated using lumped parameters. Recent developments in this area include a new local interaction simulation approach (LISA) [22–24]. This method uses no finite difference equation but simulates wave propagation heuristically, i.e. directly from physical phenomena and properties.

More recently, various spectral methods have been proposed for analysis of elastic wave propagation in complex media [25,26].

The Forward Fourier Transformation (FFT)-based Spectral Element Method proposed by Doyle [25] is very similar to the technique of the FEM as far as the assembly and the solution of the equation of motion are concerned. Firstly, the excitation signal is transformed into a number of frequency components using the FFT. Next as a part of a big frequency loop (as opposed to a loop over time in the conventional FE formulation) the dynamic stiffness matrix is generated, transformed, and next a solution is found for each unit impulse at each frequency. This yields directly to the frequency response function (FRF) of an analysed problem. The calculated frequency domain responses are then transformed back to the time domain using the Inverse Fast Fourier Transformation (IFFT). It proves that this technique is well suited for simple one-dimensional problems [27–30] but is inefficient when the geometry becomes complex or when two- or three-dimensional problems must be analysed.

The SEM as proposed by Patera in 1984 [26] is much more versatile for the analysis of elastic wave propagation in structures of complex geometry. The method originates from the use of spectral series for solution of partial differential equations [31]. The idea of this technique is very similar to the FEM. The main assumption made here is to utilise as approximation functions orthogonal Legendre or Cheybysev polynomials and to define element nodes at Gauss–Lobatto–Legendre points. As a consequence of these two factors the obtained element mass matrix is diagonal. In this way the cost of numerical calculation is much less expensive than in the case of the classic FE approach. Also thanks to the orthogonality of the approximation functions the SEM assures exponential convergence. The main fields of application of the SEM nowadays include fluid dynamics [32], heat transfer [33], acoustics [34], seismology [35], etc.

However, it appears that the use of the SEM for analysis of wave propagation in composite materials has not been reported in the literature so far. The aim of this paper is to develop a spectral membrane finite element which can next be successfully used for the analysis of in-plane elastic wave propagation in a composite panel. Numerical calculations are carried out for various fibre orientations as well as for various relative volume fractions of the fibres. Results of numerical calculations show that the velocity and direction of propagating in-plane elastic waves are functions of the orientation of the fibres and their relative volume fraction.

2. Spectral membrane finite element formulation

2.1. Definition of element nodes

In the current formulation of the spectral membrane finite element based on the SE approach the nodes of the element are defined in the local coordinate system of the element $\xi\eta$ as roots of the following polynomial expression:

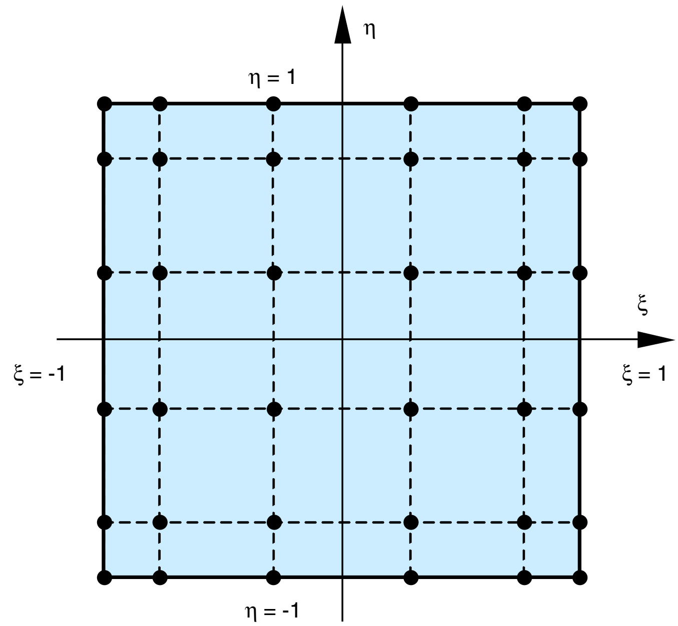

Fig. 1. A 36-node spectral finite element in the local coordinate system.

$$\begin{cases} (1-\xi)^2 P_5'(\xi) = 0, \\ (1-\eta)^2 P_5'(\eta) = 0, \end{cases} \tag{1}$$

where $\xi,\ \eta \in [-1, 1]$ and where P_5 is the fifth order Legendre polynomial. The symbol$'$ denotes the first derivative. In this way the nodes of the element can be specified in the local coordinate system of the element $\xi\eta$ as (see Fig. 1 for details)

$$\xi, \eta = \left(-1, -\sqrt{\frac{1}{3}+\frac{2}{3\sqrt{7}}}, -\sqrt{\frac{1}{3}-\frac{2}{3\sqrt{7}}}, \sqrt{\frac{1}{3}-\frac{2}{3\sqrt{7}}}, \sqrt{\frac{1}{3}+\frac{2}{3\sqrt{7}}}, 1\right) \to (\xi_m, \eta_n),$$
$$m, n = 1, \ldots, 6. \tag{2}$$

It can be seen from Fig. 1 that the present definition of the element nodes results in a irregular distribution of the nodes within the element (in the local coordinate system of the element $\xi\eta$) contrary to the classical FE approach when the element nodes are uniformly spaced within elements or on the element boundary—as in the formulation of the Lagrange or Serendip type of finite elements.

2.2. Element shape functions

A set of shape functions can be built on the specified nodes to approximate the geometry of the element in the global