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3D computational modeling of powder compaction processes using a three-invariant hardening cap plasticity model

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Abstract

In this paper, a three-invariant cap plasticity is developed for description of powder behavior under cold compaction process. The constitutive elasto-plastic matrix and its components are derived as the nonlinear functions of powder relative density. Different aspects of 2D and 3D cap plasticity models are illustrated and the procedure for determination of powder parameters is described. It is shown how the proposed model could generate the elliptical yield surface, double-surface cap plasticity and the irregular hexagonal pyramid of the Mohr–Coulomb and cone-cap yield surface, as special cases. The single-cap plasticity is performed within the framework of large finite element deformation, in order to predict the nonuniform relative density distribution during powder die pressing. Finally, the applicability of the proposed model for description of powder behavior is demonstrated in numerical simulation of triaxial and confining pressure tests. The numerical schemes are examined for efficiency in the modeling of an automotive component, a conical shaped-charge liner and a connecting-rod.

Keywords: 3D compaction model; Three-invariant plasticity; Isotropic hardening; Powder behavior

1. Introduction

Powder pressing process is the main part of many powder metallurgy manufacturing routes. The physical and mechanical properties of powder metallurgy (PM) components are closely related to their final density. Minimizing the density gradients is an important consideration when high and consistent mechanical performance is required. The final density is determined by the press-and-sinter process parameters and by the material characteristics and filling conditions. The density distribution of the material in the as-poured condition has effects that are propagated throughout the subsequent PM processes. Understanding and quantifying the main factors that influence the fill density could be a platform for 'tailoring' the initial density in the die.

Finite element simulations of the compaction process in combination with appropriate material laws for the powder allow quantitative predictions of the tool loading, of the green density distribution and of the sinter distortions. The analysis of powder pressing requires accurate material models of the various powder mixes that are used. Thus, an efficient and reliable plasticity model will play an important role in powder compaction simulation. The experimental results of Watson and Wert [1] and Brown and Abou-Chedid [2] demonstrated that the constitutive modeling of geological and frictional materials can be utilized to construct the suitable phenomenological constitutive models which capture the major features of the response of initially loose powders to the complex deformation processing histories encountered in the manufacture of powder components. In particular, they suggested that a 'two-mechanism-model', such as: Drucker–Prager or Mohr–Coulomb and elliptical cap models, which exhibit pressure dependent behavior can be useful for modeling the response of powder materials.

The cone-cap model based on a density-dependent Drucker–Prager yield surface and a noncentered ellipse is developed by Aydin et al. [3], Khoei and Lewis [4], Brandt and Nilsson [5] and Gu et al. [6]. A double-surface plasticity

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model was developed by Lewis and Khoei [7] for the nonlinear behavior of powder materials in the concept of the generalized plasticity formulation for the description of cyclic loading. This model is based on the combination of a convex yield surface consisting of a failure envelope, such as a Mohr–Coulomb yield surface and a hardening elliptical cap. The model comprises two surfaces, one to reflect shear failure and the other to capture densification. Recently, Khoei et al. [8,9] developed a density-dependent endochronic theory based on coupling between deviatoric and hydrostatic behavior in finite strain plasticity to simulate the compaction process of powder material.

Up to date, the most computational simulation of powder compaction processes has been presented in two-dimensional (2D) cases. A Lagrangian kinematic formulation was developed by Oliver et al. [10] and Lewis and Khoei [11] to model the 2D multi-level components. The *h*-adaptive FE technique was employed by Khoei and Lewis [12], in order to avoid the progressive mesh distortion during different stages of compaction. An arbitrary Lagrangian–Eulerian approach was developed by Rodriguez-Ferran et al. [13] and Khoei et al. [14] in 2D powder forming simulation, as the adaptive mesh refinement was computationally expensive and information had to be interpolated from the old mesh to the new mesh. However, to the knowledge of authors less numerical modeling has been reported in 3D powder compaction simulation. Recently, the 3D compaction simulations of geometries with circular and quadratic cross-sections were performed by Cedergren et al. [15]. Also, a 3D FE modeling of multi-level powder components was presented by Khoei and Azizi [16] using a double-surface plasticity theory.

In the present paper, a generalized three-invariant single-cap plasticity is developed for 3D simulation of powder forming processes. The cap plasticity model is developed based on the nonlinear functions of powder relative density. The constitutive elasto-plastic matrix and its components are extracted. The procedure for determination of powder parameters is described and the applicability of the model is demonstrated in several numerical examples.

2. Large finite element deformation

The nonlinearities in powder forming analyses arise from two distinct sources; constitutive nonlinearities and geometric nonlinearities, the latter being due to large displacements. Whether the displacements, or strains, are large or small it is imperative that the equilibrium conditions between the internal and external forces have to be satisfied. Thus, the equilibrium equation of a body in a deformed configuration can be written in a standard form as

$$\sigma_{ij,j} + \rho b_i = 0, \tag{1}$$

where u_i is the current displacement, b_i is the body force, ρ is the current density of powder and σ_{ij} is the total (Cauchy) stress. The constitutive law with respect to the incremental stress can be defined as

$$\mathrm{d}\sigma_{ij} = D_{ijkl}(\mathrm{d}\varepsilon_{kl} - \mathrm{d}\varepsilon^0_{kl}) + \sigma_{ik}\mathrm{d}\omega_{kj} + \sigma_{jk}\mathrm{d}\omega_{ki}, \tag{2}$$

where $\mathrm{d}\epsilon_{ij}$ and $\mathrm{d}\omega_{kl}$ are the incremental values of the strain and rotation, respectively. The last two terms account for the Zaremba–Jaumann rotational stress changes (negligible generally in small displacement computation). In Eq. (2), D_{ijkl} is a tangential matrix, defined by suitable state variables and the direction of the increment, and ε^0_{ij} refers to strains caused by external actions such as temperature changes.

For geometrically nonlinear behavior, we can select either a total or an updated Lagrangian coordinate system. If the initial undeformed position of a particle of material is $\mathbf{x_0}$ and the total displacement vector is $\mathbf{u}$ then the coordinates of the particle are $\mathbf{x} = \mathbf{x_0} + \mathbf{u}$. A general definition of strains, which is valid whether the displacements or strains are large or small, was introduced by Green and St. Venant. Based on Green's strain tensor, the nonlinear strain displacement relationship can be defined in terms of the infinitesimal and large displacement components as

$$\boldsymbol{\epsilon} = \boldsymbol{\epsilon}_{\mathrm{L}} + \boldsymbol{\epsilon}_{\mathrm{NL}} = \boldsymbol{\epsilon}_{\mathrm{L}} + \tfrac{1}{2}\boldsymbol{A}_\theta\boldsymbol{\theta}, \tag{3}$$

where $\boldsymbol{\epsilon}_{\mathrm{L}}$ and $\boldsymbol{\epsilon}_{\mathrm{NL}}$ are the linear and nonlinear strains and for 3D problems are defined as

$$\boldsymbol{\epsilon}_{\mathrm{L}} = \begin{Bmatrix} \dfrac{\partial u}{\partial x} \\ \dfrac{\partial v}{\partial y} \\ \dfrac{\partial w}{\partial z} \\ \dfrac{\partial v}{\partial x} + \dfrac{\partial u}{\partial y} \\ \dfrac{\partial w}{\partial y} + \dfrac{\partial v}{\partial z} \\ \dfrac{\partial u}{\partial z} + \dfrac{\partial w}{\partial x} \end{Bmatrix},$$

$$\boldsymbol{\epsilon}_{\mathrm{NL}} = \begin{Bmatrix} \frac{1}{2}\left(\dfrac{\partial u}{\partial x}\right)^2 + \frac{1}{2}\left(\dfrac{\partial v}{\partial x}\right)^2 + \frac{1}{2}\left(\dfrac{\partial w}{\partial x}\right)^2 \\ \frac{1}{2}\left(\dfrac{\partial u}{\partial y}\right)^2 + \frac{1}{2}\left(\dfrac{\partial v}{\partial y}\right)^2 + \frac{1}{2}\left(\dfrac{\partial w}{\partial y}\right)^2 \\ \frac{1}{2}\left(\dfrac{\partial u}{\partial z}\right)^2 + \frac{1}{2}\left(\dfrac{\partial v}{\partial z}\right)^2 + \frac{1}{2}\left(\dfrac{\partial w}{\partial z}\right)^2 \\ \dfrac{\partial u}{\partial x}\dfrac{\partial u}{\partial y} + \dfrac{\partial v}{\partial x}\dfrac{\partial v}{\partial y} + \dfrac{\partial w}{\partial x}\dfrac{\partial w}{\partial y} \\ \dfrac{\partial u}{\partial y}\dfrac{\partial u}{\partial z} + \dfrac{\partial v}{\partial y}\dfrac{\partial v}{\partial z} + \dfrac{\partial w}{\partial y}\dfrac{\partial w}{\partial z} \\ \dfrac{\partial u}{\partial z}\dfrac{\partial u}{\partial x} + \dfrac{\partial v}{\partial z}\dfrac{\partial v}{\partial x} + \dfrac{\partial w}{\partial z}\dfrac{\partial w}{\partial x} \end{Bmatrix}. \tag{4}$$

In small displacement theory, the general first-order linear strain approximation is obtained by neglecting the quadratic terms. In Eq. (3), the nonlinear terms of strain $\boldsymbol{\epsilon}_{\mathrm{NL}}$ is defined as $\boldsymbol{\epsilon}_{\mathrm{NL}} = \frac{1}{2}\boldsymbol{A}_\theta\boldsymbol{\theta}$, with $\boldsymbol{\theta}$ denoting the displacement gradient and $\boldsymbol{A}_\theta$ a