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Robust Power Control for Cognitive Radio Networks with Proportional Rate Fairness

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Abstract

This paper studies the power control problem in cognitive radio networks where a primary user and multiple secondary users (SUs) coexist. Imperfect channel state information is considered. The objective is to maximize the SUs' sum rate while guaranteeing the proportional rate fairness among SUs. The problem under consideration is non-convex. By doing a transformation, it is equivalently changed to a second-order cone programming problem, which can be efficiently solved by existing standard methods. Simulations have been done to verify the network performance under different channel uncertainty conditions.

Index Terms: Cognitive radio networks, Robust power control, Imperfect channel state information, Proportional rate fairness

1. Introduction

As the rapid development of advanced technologies on wireless communications, a lot of high transmission rate services and applications have emerged, which increases the demand for spectrum. On the other hand, experimental results have shown that traditional fixed spectrum allocation schemes yield inefficient spectrum utilization [1]. To improve the spectrum utilization and provide high quality of services (QoS), cognitive radio networks (CRNs) that allow the unlicensed secondary users (SUs) share the licensed spectrum with the licensed primary users (PUs) have been proposed.

Spectrum allocation problem in CRNs has drawn large attention in recent years [2-7]. In most of these works, it is assumed that perfect channel state information (CSI) is known [2-4]. However, in practice perfect CSI, especially the channel gain from the SUs to PUs, cannot be obtained due to the lack of cooperation among PUs and SUs. Therefore, this motivates the research on resource allocation problem in CRNs with imperfect CSI [4-6]. Mitliagkas et al. investigated the joint power control and admission control problem in [5]. Kim et al. in [6] studied the sum rate maximization problem under the total power and interference power constraints. Parsaeefard et al. in [7] worked on the social utility of SUs while satisfying each SU's signal to noise ratio requirement and interference power constraint. However, all those works do not explicitly consider SUs' different transmission rate requirements and fairness issue, thus they are not suitable for a situation where different SUs have different transmission rate requirements. To flexibly allocate transmission rates to each SU and guarantee fairness among SUs, we will investigate the resource allocation problem with proportional rate fairness requirements in CRNs under imperfect CSI.

In this paper, we will investigate the power control problem in CRNs, where imperfect CSI from secondary BS to the primary user is considered. The objective is to maximize the SUs' sum rate subject to the proportional rate fairness constraint among SUs, the total power constraint at secondary BS, and the interference power constraint to the PU. The problem is formulated as a non-convex optimization problem. By doing a transformation, the problem is changed to an equivalent second-order cone programming (SOCP) problem, which can be efficiently solved by existing standard methods. Simulations have been done to demonstrate the network performance under different channel uncertainty conditions.

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2. System Model and Problem Formulation

Consider a network setting where a PU and K SUs coexist. Downlink transmission from the secondary base station (BS) to SUs is considered. The SUs can adopt the available channels that are licensed to the PU for its own data transmission. It is assumed that the total available bandwidth is divided into multiple non-overlapping channels. And each SU is allocated one such channel for its own data transmission.

The channel gain from the secondary BS to SU k, $\forall k \in \{1,2,\cdots,K\}$ is denoted by h_k . σ_k is the variance of the additive white Gaussian noise in that channel. For notational brevity, let $H_k = h_k / \sigma_k$. The data rate for SU k isdenoted by

$$R_k = 0.5\log_2\left(1 + H_k P_k\right), \tag{1}$$

where P_k is the transmission power for SU k at BS.

To protect the PU's QoS, the interference to the PU should not be greater than the given threshold T_{th} , which can be expressed by

$$\sum_{k=1}^{K} P_k d_k \le T_{th} , \tag{2}$$

Where d_k is the channel gain for SU k from the secondary BS to the primary user. In practice, imperfect channel information cannot be obtained, especially the channel gain from the secondary users to the primary users. Because generally there is a lack of cooperation between primary user and SUs, and thus the primary user will not feedback the CSI to the SUs. Ellipsoidal uncertainty will be adopted to model the uncertainty of channel gain d_k . Let us define vector $\mathbf{d} = \left[d_1 \quad d_2 \quad \cdots d_K\right]^T$. Adopting the ellipsoidal uncertainty [5], the uncertainty region of $\mathbf{d}$ can be expressed by

$$\mathrm{d} \in \Omega = \left\{\bar{\mathrm{d}} + \mathrm{Du} : \|\mathrm{u}\|_2 \le 1\right\}, \tag{3}$$

where $\bar{\mathbf{d}}$ is the nominal value of $\mathbf{d}$, D is a $K \times K$ matrix, and u is a K dimensional vector. To facilitate the following analysis, let us define a vector $\mathbf{P}_s = \left[P_1 \quad P_2 \quad \cdots \quad P_K\right]^T$, and then (2) can be rewritten as

$$\mathbf{d}^T \mathbf{P}_s \le T_{th} . \tag{4}$$

Since $\mathbf{d}$ satisfies (3), to guarantee (4) hold, it is equivalent to make sure the following inequality (5) holds,

$$\sup\left\{\mathbf{d}^T \mathbf{P}_s \middle| \mathbf{d} \in \Omega\right\} \le T_{th} . \tag{5}$$

From (5), by invoking the Cauchy-Schwarz inequality, one gets that

$$\begin{aligned} &\sup\left\{\mathbf{d}^T \mathbf{P}_s \middle| \mathbf{d} \in \Omega\right\} \\ &= \bar{\mathbf{d}}^T \mathbf{P}_s + \sup\left\{\mathbf{u}^T \mathbf{D}^T \mathbf{P}_s \middle| \|\mathbf{u}\|_2 \le 1\right\}. \\ &\le \bar{\mathbf{d}}^T \mathbf{P}_s + \|\mathbf{u}\|_2 \left\|\mathbf{D}^T \mathbf{P}_s\right\|_2 \\ &= \bar{\mathbf{d}}^T \mathbf{P}_s + \left\|\mathbf{D}^T \mathbf{P}_s\right\|_2 \le T_{th} \end{aligned} \tag{6}$$

We desire to study the power control problem to maximize the sum rate of SUs under several constraints. The problem under consideration can be formulated as follows,

$$\begin{aligned} &\max_{\{P_k, R_k\}} \sum_{k=1}^{K} R_k \\ &s.t. \begin{cases} C1. \sum_{k=1}^{K} P_k \le P_{th} \\ C2. 0 \le P_k, \forall k \in \{1,2,\cdots,K\} \\ C3.(6) \\ C4. R_1 : R_2 : \cdots : R_K = \gamma_1 : \gamma_2 : \cdots : \gamma_K \\ C5.(1), \forall k \in \{1,2,\cdots,K\} \end{cases} \end{aligned} \tag{7}$$

Where $C1$ represents the BS total power constraint, and P_{th} is the power threshold at the BS. $C2$ indicates that the consumed power for each SU at the BS should be non-negative. $C3$ is the interference power constraint to the primary user. $C4$ is the proportional rate fairness constraint; $\gamma_1, \gamma_2, \cdots, \gamma_K$ are given constants, and they indicate the proportional rate requirements of SUs. $C5$ represents the SU's transmission rate constraint.

3. Optimal Solution

Problem (7) is a non-convex optimization problem since the nonlinear equality constraint $C5$. To make the problem easy to solve, we will transform problem (7) into its equivalent form.

By replacing the equality constraint in $C5$ by an inequality constraint

$$R_k \le 0.5\log_2\left(1 + H_k P_k\right), \tag{8}$$

problem (7) becomes

$$\begin{aligned} &\max_{\{P_k, R_k\}} \sum_{k=1}^{K} R_k \\ &s.t. \begin{cases} C1 \sim C4. \\ C5'. R_k \le 0.5\log_2\left(1 + H_k P_k\right), \forall k \in \{1,\cdots,K\} \end{cases} \end{aligned} \tag{9}$$

Problem (9) is an SOCP problem, since its objective function is a linear function, its constraint set is a convex set, and $C3$ is a second-order cone constraint. A proposition will be given in the following to show that the optimal solution of problem (9) satisfies $R_k = 0.5\log_2\left(1 + H_k P_k\right)$, and thus problem (9) is equivalent to problem (7). Hence, we can solve Problem (9) instead of Problem (7).

Proposition 1. The rates that optimize problem (9) satisfy that $R_k = 0.5\log_2\left(1 + H_k P_k\right)$, $\forall k \in \{1,2,\cdots,K\}$.

Proof. Because the objective function of problem (9) is an increasing function with respect to R_k , and R_k satisfies constraint $C5$'. It is easy to see that when problem (9) admits its optimal solution R_k satisfies that $R_k = 0.5\log_2\left(1 + H_k P_k\right)$, $\forall k \in \{1,2,\cdots,K\}$. ∎