ICT Express
Information & Communications Technology

Generation of Binary Off-axis Digital Fresnel Hologram with Enhanced Quality

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Abstract

The emergence of high resolution printer and digital micromirror device (DMD) has enabled real, off-axis holograms to be printed, or projected onto a screen. As most printers and DMD can only reproduce binary dots, the pixels in a hologram have to be truncated to 2 levels. However, direct binarizing a hologram will lead to severe degradation on its reconstructed image. In this paper, a method for generating binary off-axis digital Fresnel hologram is reported. A hologram generated with the proposed method is referred to as the "Enhanced Sampled Binary Hologram" (ESBH). The reconstructed image of the ESBH is superior in visual quality as compare with the one obtained with existing technique, and also resistant to noise contamination.

Index Terms: Binary off-axis hologram, Grid-cross down-sampling, Enhanced sampled binary hologram

1. Introduction

A hologram is a complex image which, when illuminated with a coherent beam, will reconstruct an observable three-dimensional object scene with full depth and disparity information. Modern research has also shown that a hologram can be computed numerically and stored in a digital file [1]. It will be desirable if the digital hologram can be reproduced on a physical media through a commodity printer [2], or a digital micromirror device (DMD) [3]. However, it is well known that commodity printers and DMD can only write real value pixels, and generally as binary (i.e. black and white) dots. An effective solution to address this issue, which is less complicated than using multiplexing multiple diffusers [3], is to convert the complex hologram into a real image, and quantizing the intensity of the each pixel intensity into a binary value. The above process can be briefly described as follows. To start with, consider an object scene with the brightness of each object point denoted by the intensity image. The perpendicular distance of an object point to the hologram is represented by the depth map. The complex hologram can be generated from the scene according to the Fresnel diffraction equation as [1]

$$H(u,v) = \sum_{x=0}^{X-1}\sum_{y=0}^{Y-1} \frac{I(u,v)\exp\left(i2\pi r_{u;v;x;y}/\lambda\right)}{r_{u;v;x;y}}, \qquad (1)$$

where $r_{u;v;x;y} = \sqrt{(x-u)^2\delta^2 + (y-v)^2\delta^2 + w_{u;v}^2}$ is the Euclidean distance between an object point at co-ordinate (x,y) in the 3D scene, and a pixel at (u,v) on the hologram. λ is the wavelength of the optical beam. The hologram and the object scene are assumed to have the same horizontal and vertical extents, comprising of X columns and Y rows of pixels. The complex hologram derived from Eq. (1) can be converted into a real off-axis digital Fresnel hologram $H_1(u,v)$ by multiplying the hologram with an inclined planar wave $R(v)$, and dropping the imaginary part, i.e.,

$$H_1(u,v) = \mathrm{Re}\left[H(u,v)R(v)\right]. \qquad (2)$$

Subsequently, the hologram $H_1(u,v)$ can be binarized with sign-thresholding, setting 1 (white) and 0 (black) values to pixels of positive and negative polarity, respectively. However, direct binarizing a hologram with sign-thresholding will lead to severe degradation on shaded (i.e. smooth or homogeneous) region of the reconstructed

Received 25 August 2014; **Revised** 8 September 2014 **Accepted** 20 September 2014

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image. Although a hologram can be binarized with iterative means [4][5], the computation could be rather intensive. To overcome this problem, Tsang et al [6] had proposed to down-sampled the intensity image with a lattice of grid-cross patterns before generating the hologram. The visual quality of the reconstructed image derived from the down-sampled image is favorable, but the shaded region is masked with a coarse granular pattern. In this paper, a method to address the above-mentioned problem is proposed. An off-axis hologram generated with the proposed scheme is referred to as the "Enhanced Sampled Binary Hologram" (ESBH). The visual quality of the reconstructed image of a ESBH is superior to the one generated with the existing methods, and also resistant to noise contamination. In the remaining parts of this paper, detail of the proposed method is presented in Section 2. Experimental evaluation is provided in Section 3, and a conclusion will be given at the end of the paper summarizing the essential findings.

2. Proposed method for generation of the Enhanced Sampled Hologram

For the sake of clarity of explanation, the concept for generating binary off-axis Fresnel hologram through down-sampling is shown in Fig. 1, and outlined as follows. To begin with, an object scene is modeled by an intensity image $I(x,y)$ and a depth map $D(x,y)$. The intensity image is first down-sampled by a lattice $S(x,y)$ as given by

$$I_d(x,y) = I(x,y) \times S(x,y). \tag{3}$$

The lattice $S(x,y)$ is a two-dimensional image with the sample points set to unity, and the non-sample points set to zero. Next, Eq. (1) is applied to convert the down-sampled image $I_d(x,y)$ and the depth map $D(x,y)$ into a complex hologram $H(x,y)$. The latter is multiplied with a reference plane wave $R(v)$, and the real part of the product is quantized to a binary off-axis hologram $H_b(x,y)$ through sign-thresholding. In the method reported in [6], a grid-cross down-sampling lattice with the sampling points positioned along regularly spaced horizontal, vertical, and diagonal lines is employed. The reconstructed image of the grid-cross sampled hologram has favorable visual quality, and capable of preserving the intensity distribution at the shaded areas. However, the intensity at the crossing points of the sampled lines is found to be relatively stronger than the rest of the areas, resulting to a granular and unnatural appearance. To overcome the above mentioned problem, a new down-sampling scheme is proposed in this paper. In essence, a uniform grid down-sampling lattice with the sample points removed near the crossing points of the line segments is adopted. Mathematically, the new down-sampling lattice can be expressed as

$$S(x,y) = S_c(x,y) \times N(x,y). \tag{4}$$

In Eq. (4), $S_c(x,y)$ is a grid down-sampling lattice comprising of regular spaced horizontal and vertical lines as given by

$$S_c(x,y) = \begin{cases} 1 & (x \bmod M) \times (y \bmod M) = 0 \\ 0 & \textit{otherwise} \end{cases}, \tag{5}$$

where M is the down-sampling factor. The other term $N(x,y)$ is a two-dimensional mask where all the entries within a $\tau \times \tau$ square region centered at each crossing point of the grid down-sampling lattice is set to zero, and the remaining elements are all set to unity. A small section of the grid sampling lattice $S_c(x,y)$ is shown in Fig. 2(a) to illustrate the formation of the new down-sampling lattice $S(x,y)$. The mask $N(x,y)$ is shown in Fig. 2(b), and the new down-sampling lattice that is derived from Eq. (4) is shown in Fig. 2(c). It can be seen that $S(x,y)$ is comprising of regular spaced horizontal and vertical lines without the crossing points. As will be shown in the later part of this paper, the elimination of the sampling points around the crossing points will reduce the granular effect, resulting in a more natural visual quality on the reconstructed image of the binary hologram.

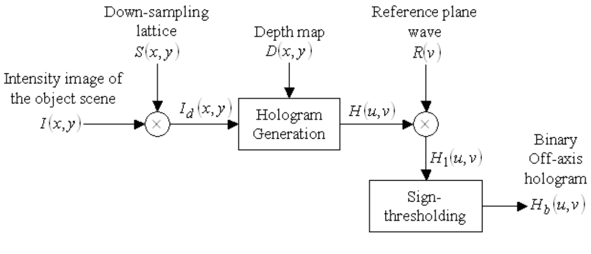

Fig. 1 Method for generating binary off-axis Fresnel hologram

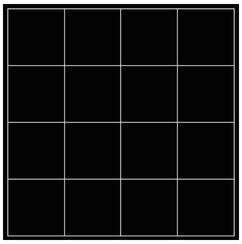

Fig. 2(a) The grid sampling lattice $S_c(x,y)$