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A fractional calculus approach to modeling rheological behavior of soft magnetic elastomers

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ABSTRACT

A fractional calculus approach is applied to model constitutive behavior of magnetorheological elastomers. To test the model results magnetorheological elastomers containing different concentrations of iron particles have been synthesized and their rheological behavior in various magnetic fields has been studied experimentally. The fractional model parameters were found from fitting the experimental data. An excellent agreement between experimental results and the model predictions has been obtained. Correlation between the values of the order of the constitutive equation and structuring of magnetic filler governed by the balance between elastic and magnetic interactions has been formulated. It has been shown that the fractional rheological model can satisfactorily describe dynamic behavior of magnetorheological elastomers in magnetic field.

1. Introduction

Soft magnetic elastomers or magnetorheological elastomers (MREs) belong to the class of smart materials being able to change their properties considerably in applied magnetic fields [1–3]. MRE consists of a polymer matrix with embedded magnetic particles. In an external magnetic field the magnetic particles start to interact, and can even restructure if the polymer matrix is soft enough [4,5], these interactions resulting in a change of various physical properties of MREs, in particular, their mechanical characteristics [6–17] as well as electromagnetic properties [17–21]. The detailed analysis of the recent progress in development of MREs including their composition, synthesis and functional behavior can be found in reviews [1–3].

Owing to the most straightforward practical applications of MREs as tunable dampers or vibration absorbers most of the studies have mainly been focused on viscoelastic behavior of MREs [5–17]. It has been shown experimentally in numerous publications that MREs can reversibly change their static shear and Young's moduli by several orders of magnitude [1,7,8,10] in uniform magnetic fields. Besides, a lot of attention has been paid recently to the dynamic mechanical behavior of MREs and a considerable increase of the MRE complex dynamic modulus (the storage G' and the loss G'' moduli) in external magnetic field has also been reported [11–17]. The dynamic behavior is of especial significance for practice when MREs are exposed to external oscillating mechanical force. In particular, the damper behavior is mainly defined by rheological properties of the viscoelastic material they are made of.

In spite of extensive experimental work unified and consistent theoretical modeling of dynamic behavior of MREs in magnetic fields is still missing. Over the years, different approaches to describe magnetic response of MREs in static loading experiments were undertaken. Most notable patterns include continual approach [22–27] and microscopic theories [28–33]. Concerning dynamic properties, they were modeled via building viscoelastic constitutive relation of MREs with the use of the classical viscoelastic model schemes of macroscopic rheology consisting of springs and dashpots [34–37]. Phenomenological schemes do not have strict microscopic justification, suitability of the model is characterized by its ability to fit well with experimental data [38]. Owing to the complex behavior of multi-component MREs in magnetic fields a good fitting can be achieved by addition of rheological elements, this in turn considerably complicates the analysis.

In this paper, we propose a fractional rheological model to simulate MREs dynamic response to magnetic field. Fractional rheological models are successfully applied nowadays to describe fundamental properties of different materials, in particular,

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rheological behavior of linear viscoelastic media [39–44]. Furthermore, there were a few attempts to simulate magneto-dependent damping properties of MREs with the use of a fractional element in rather complicated rheological schemes [45,46]. In the present paper we propose a simple model based on the classical Poynting–Thomson scheme and first pay close attention to its parameters and their behavior in the presence of a uniform magnetic field. We try to understand the connection between changes of MRE structure in magnetic field and fractional rheological models. We provide additional proof of fractional approach usefulness and take a first step in constructing an optimal fractional rheological model for MREs.

The paper is organized as follows. In the following section the classical Poynting–Thomson model is described, then the fractional Poynting–Thomson model is formulated and its constitutive equation is derived. In Section 3 we briefly outline the basic definitions of fractional calculus. Section 4 gives some details of the experiments, it is followed by model fitting the experimental frequency dependences of the storage and loss moduli. Finally, in Section 6 we discuss the effect of magnetic field on the model parameters. The results are summarized in Conclusion. In Appendix it is demonstrated that the proposed fractional rheological model does catch some important properties of the material, namely stress-strain hysteresis.

2. Rheological model

Rheological models describing the dynamic response of viscoelastic systems consist of different mechanical elements such as springs and dashpots. Each of these elements has a different dynamic law and corresponds to different processes occurring in the material on the microscale. By combining them in a ways similar to electric circuits one can get various stress-strain relations (see, for instance, [38]). Let us remind the results of a classical Poynting–-Thomson model, its rheological scheme is shown in Fig. 1.

In Fig. 1 G_1 and G_2 springs describe elastic properties, while η dashpot accounts for viscosity. This is a three-parameter model. A constitutive equation for PT model is derived in the following way. Let us denote the stress and the strain in the upper and lower branches of the model as (σ_1,ε_1) and (σ_2,ε_2) respectively. Let (σ_{el},ε_{el}) and (σ_{visc},ε_{visc}) be the stresses and the strains for the G_2 spring and for the dashpot respectively.

Let us suppose the strain is small enough for the strings to be Hookean. The dashpot is a classical Newtonian element. Thus, the stress-strain relations for these elements read:

$$\sigma_1(t) = 2G_1 \cdot \varepsilon_1(t)$$

$$\sigma_{el}(t) = 2G_2 \cdot \varepsilon_{el}(t)$$

$$\sigma_{visc}(t) = 2\eta \cdot \frac{d\varepsilon_{visc}(t)}{dt}$$

According to the analogy with an electric circuit, stress plays the role of the “electric current” while strain corresponds to “voltage”. Keeping that in mind, we can easily derive the following relations: $\sigma=\sigma_1+\sigma_2$ for the total stress, $\varepsilon=\varepsilon_1=\varepsilon_2$ for the total strain with-$\sigma_2=\sigma_{el}=\sigma_{visc}$ and $\varepsilon_2=\varepsilon_{el}+\varepsilon_{visc}$. Differentiating ε_2 we obtain:

$$\dot{\varepsilon} = \frac{d}{dt}(\varepsilon_{el} + \varepsilon_{visc}) = \frac{\sigma_2}{2\eta} + \frac{\dot{\sigma}_2}{2G_2}$$

Then we express σ_2 in terms of the total strain and stress:

$$\sigma = \sigma_1 + \sigma_2 = 2G_1\varepsilon + \sigma_2 \Rightarrow \sigma_2 = \sigma - 2G_1\varepsilon$$

Finally, by substituting σ_2 we obtain the constitutive equation:

$$\frac{1}{2G_2}\dot{\sigma} + \frac{1}{2\eta}\sigma = \left(1 + \frac{G_1}{G_2}\right)\dot{\varepsilon} + \frac{G_1}{\eta}\varepsilon. \tag{1}$$

This equation is a stress-strain relation for the classical PT model.

The simplest PT model does not adequately describe rheological properties of MREs because of their structural complexity. This model could be modified to catch some of MRE features. For instance, in Ref. [34] a combination of two classical PT chains was used to obtain some meaningful results. Such approach based on the enhancement of the number of classical elements in a chain and their rearranging, however, leads to serious complications for analytical calculations while still not being able to cover dynamics of MREs in magnetic fields.

In the present work, we propose a generalized version of Poynting–Thomson model which utilizes a more advanced mathematical apparatus but has a simple structure. The main improvement is the substitution of the dashpot by a fractional element combining material elastic and viscous properties. We call this modified model a fractional Poynting–Thomson model (FPT), it is shown in Fig. 2.

The fractional element in FPT model is based around the fractional derivative operator. It is a special differintegral representing a derivative of a fractional order. A detailed overview of fractional operators and their properties is given in the next section. The stress-strain relation for the fractional element has the form: $\sigma_{frac}(t) = c \cdot {}_a^R D_t^\alpha \varepsilon_{frac}(t)$, where c is a viscoelasticity factor, ${}_a^R D_t^\alpha$ is a

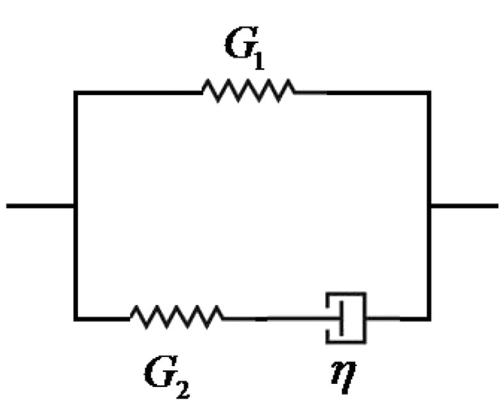

Fig. 1. Classical Poynting–Thomson model (PT).

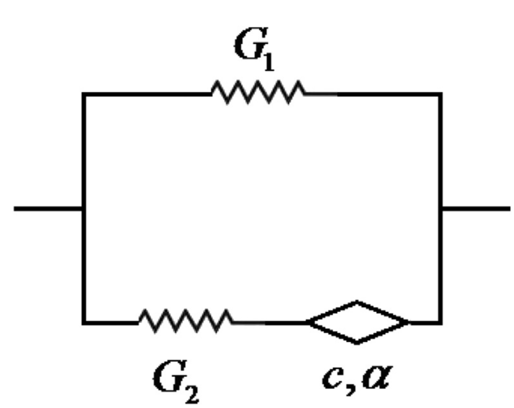

Fig. 2. Fractional Poynting–Thomson model (FPT).