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The parallel subdomain-levelset deflation method in reservoir simulation

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ABSTRACT

Extreme and isolated eigenvalues are known to be harmful to the convergence of an iterative solver. These eigenvalues can be produced by strong heterogeneity in the underlying physics. We can improve the quality of the spectrum by 'deflating' the harmful eigenvalues. In this work, deflation is applied to linear systems in reservoir simulation. In particular, large, sudden differences in the permeability produce extreme eigenvalues. The number and magnitude of these eigenvalues is linked to the number and magnitude of the permeability jumps. Two deflation methods are discussed. Firstly, we state that harmonic Ritz eigenvector deflation, which computes the deflation vectors from the information produced by the linear solver, is unfeasible in modern reservoir simulation due to high costs and lack of parallelism. Secondly, we test a physics-based subdomain-levelset deflation algorithm that constructs the deflation vectors a priori. Numerical experiments show that both methods can improve the performance of the linear solver. We highlight the fact that subdomain-levelset deflation is particularly suitable for a parallel implementation. For cases with well-defined permeability jumps of a factor 10^4 or higher, parallel physics-based deflation has potential in commercial applications. In particular, the good scalability of parallel subdomain-levelset deflation combined with the robust parallel preconditioner for deflated system suggests the use of this method as an alternative for AMG.

1. Introduction

Recent challenges in the petroleum industry include managing larger data sets, providing higher field resolutions and computing more accurate multiphase flow predictions. These challenges entail complex geometries and high physical contrasts in the geological formations of petroleum reservoirs. At the same time, advancements in hardware, such as (cheap) parallel systems and GPU-acceleration, demand the development of new algorithms that utilize hardware innovations to exceed previous performance records. The continuous interplay between computational demand and supply has fueled the development of advanced reservoir simulation software.

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At the core of any reservoir simulator is the solver mechanism. Modern reservoir simulators typically employ the Newton–Raphson method to solve the non-linear governing equations for a given timestep. The corresponding Jacobian matrix and linear system are solved by the Flexible Generalized Minimum Residual Method (FGMRES) [31] preconditioned by the Constrained Pressure Residual (CPR) preconditioner [3,49,50]. CPR decouples the linear system into two sets of equations, exploiting the specific properties of the pressure equation and transport equations. The former is solved with an Algebraic Multigrid (AMG) preconditioner [30], while the fully coupled system is solved using an ILU preconditioner. In this paper, the potential of an alternative for AMG only preconditioner is investigated by combining deflation method with different preconditioners (e.g., Jacobi). By removing unfavorable eigenvalues from the spectrum of the linear system, deflation can be used to improve convergence.

AMG is currently an industry standard for solving elliptic or parabolic partial differential equations. The method is robust and scalable for a fixed problem size per processor. For a fixed total problem size, however, AMG is difficult to scale. In practice, creating reliable simulations with an increased number of grid cells is expensive. Therefore, while the number of available processors increases, reservoir engineers will often work with existing solution strategies due to scalability issues. Despite the fact that AMG is optimal for serial computations of the pressure equation, the lack of strong scalability fuels the continued interest in alternatives for AMG. As a result, two-level multiscale solvers (MS) were developed over the past decade in order to construct an accurate coarse-scale system honoring the fine-scale heterogeneous data (see, e.g., [7,10,13,15,16,51]). The multiscale coarse-scale system is governed on the basis of locally computed basis functions, subject to reduced-dimensional boundary conditions and zero right-hand-side (RHS) terms. Multiscale solvers are naturally scalable and can be used as preconditioner. Combining this method with deflation strategy (which has a lower algebraic complexity and is inexpensive to set up) may lead to a robust alternative of AMG. The preferred method of deflation in this paper (subdomain-levelset deflation) is also (strongly) scalable which will be combined with Jacobi and AMG preconditioners. The combination of multiscale solver and deflation method is subject of future research.

Deflation was first proposed for symmetric linear systems and the conjugate gradient method (see e.g. [14]) by Nicolaides [29] and Dostál [9]. Both construct a deflation subspace consisting of deflation vectors to deflate unfavorable eigenvalues from the linear system. A range of deflation algorithms have been developed since, differing primarily in the method of application of the deflation operator and the approach to construct the deflation vectors. Deflation has been used with excellent results in a large number of applications, including electromagnetics [8], bubbly flow [25,34–36], structural mechanics and composite materials [17–19,24], unsteady turbulent airfoil problems [4] and wave models in ship simulations [42]. The work by Vuik and co-authors on layered problems in reservoir simulation [44–48] is the foundation for this paper.

In [22], the authors interweave algebraic multigrid cycles with deflation to solve several cases characterized by high permeability contrasts. The results are encouraging, showing improved convergence rates. We will argue, however, that Harmonic Ritz deflation, as used in [22], is unfeasible in commercial applications due to the number of iterations required to compute the deflation vectors. Another popular approach is to combine deflation with a preconditioner based on the partial solution in a two-stage method. In [1], the partial solution is obtained in high permeability regions. In a related approach, the authors in [21] solve in aggregates of nodes with similar connectivity strength. Similar to our experiences, these methods work best for high physical contrasts.

Central to the investigation by Vuik and co-authors is the relation between the occurrence of extreme eigenvalues and large jumps in the PDE coefficients. In [45], the number of extreme eigenvalues is proven to be equal to the number of high-permeability layers (e.g. sand) between low-permeability layers (e.g. shale) for the diagonally scaled system matrix. Having observed this, the question arises how to utilize the predictable spectrum in layered problems. In [45] and subsequent work, it is shown that the subspace spanned by the eigenvectors corresponding to the extreme eigenvalues can be approximated by a pre-determined space of algebraic deflation vectors. Convergence of the deflated CG method is shown to be independent of the size of the jumps in the coefficients.

We extend the work by Vuik and co-authors to non-symmetric linear systems arising from the fluid flow in porous media. Our serial physics-based deflation approach is based on the levelset deflation method [36]. Regions of approximately constant permeability, separated by large jumps, are identified, and used to construct the deflation vectors. These vectors prove to be good approximations of the eigenvectors corresponding to the extreme eigenvalues caused by the jumps. Moreover, we will argue that the levelset deflation method allows for an efficient parallel implementation. Used in parallel, our deflation algorithm becomes very similar to the subdomain-levelset deflation method [36]. In parallel subdomain-levelset deflation, the levelset deflation method is applied to each parallel subdomain. We extend the work on parallelizing the subdomain deflation method in [12,37] to parallelize the subdomain-levelset deflation method. We use numerical experiments for cases with varying size and degree of complexity to compare the performance of Harmonic-Ritz eigenvector deflation and subdomain-levelset deflation.

In the first part of this paper we give a brief introduction to deflation theory. Subsequently we present and motivate the choices of the deflation vectors and provide several numerical experiments on real simulation cases. In the last part of this paper we summarize and discuss future work.

2. Reservoir simulation

Physical properties are captured in the coefficients of the reservoir equations. We mainly focus on the permeability, or, roughly, the ease with which a fluid can flow through the porous medium. The grid, coefficients and wells give rise to a