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An asymptotic preserving unified gas kinetic scheme for frequency-dependent radiative transfer equations

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ABSTRACT

This paper presents an extension of previous work (Sun et al., 2015 [22]) of the unified gas kinetic scheme (UGKS) for the gray radiative transfer equations to the frequency-dependent (multi-group) radiative transfer system. Different from the gray radiative transfer equations, where the optical opacity is only a function of local material temperature, the simulation of frequency-dependent radiative transfer is associated with additional difficulties from the frequency-dependent opacity. For the multiple frequency radiation, the opacity depends on both the spatial location and the frequency. For example, the opacity is typically a decreasing function of frequency. At the same spatial region the transport physics can be optically thick for the low frequency photons, and optically thin for high frequency ones. Therefore, the optical thickness is not a simple function of space location. In this paper, the UGKS for frequency-dependent radiative system is developed. The UGKS is a finite volume method and the transport physics is modeled according to the ratio of the cell size to the photon's frequency-dependent mean free path. When the cell size is much larger than the photon's mean free path, a diffusion solution for such a frequency radiation will be obtained. On the other hand, when the cell size is much smaller than the photon's mean free path, a free transport mechanism will be recovered. In the regime between the above two limits, with the variation of the ratio between the local cell size and photon's mean free path, the UGKS provides a smooth transition in the physical and frequency space to capture the corresponding transport physics accurately. The seemingly straightforward extension of the UGKS from the gray to multiple frequency radiation system is due to its intrinsic consistent multiple scale transport modeling, but it still involves lots of work to properly discretize the multiple groups in order to design an asymptotic preserving (AP) scheme in all regimes. The current scheme is tested in a few frequency-dependent radiation problems, and the results are compared with the solutions from the well-defined implicit Monte Carlo (IMC) method. The UGKS is much more efficient than IMC, and the computational times of both schemes for all test cases are listed. The UGKS seems to be the first discrete ordinate method (DOM) for the accurate capturing of multiple frequency radiative transport physics from ballistic particle motion to the diffusive wave propagation.

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1. Introduction

To solve the radiative transfer equations is very challenging, but it is important in astrophysics, inertial confinement fusion, and high temperature flow systems. Due to the complexity of the system, its study attracts continuous attention from national laboratories and academic institutes. This paper presents a continuous effort to develop a useful and reliable computational method for multiple scale radiative transport system.

In a previous work, we have developed an asymptotic preserving unified gas kinetic scheme (UGKS) for the gray radiative transfer equations [22]. Here an extension of the scheme to frequency-dependent radiative transfer system will be developed. The radiative transfer equations are the modeling equations in the kinetic level, where the photon transport and collision with material are taken into account. This system can present different limiting solutions with the changing of the scales. For the gray radiative transfer equations, the opacity is just a function of the material temperature. Therefore, the spatial cells can be classified as optical thick and optical thin regions, and a domain decomposition method with different numerical discretization in different regions can be developed. However, for the frequency-dependent radiative transfer equations, the opacity is typically a decreasing function of frequency. A spatial region can be optically thick for low frequency photon, but optically thin for high frequency ones. It becomes challenging to develop a reliable asymptotic preserving scheme for simulating different frequency photon transport efficiently.

The radiative transfer equations model the radiation intensity transport and energy exchange with the background material. The properties of the background material influence greatly on the behavior of radiation transfer. For a low opacity (background) material, the interaction between the radiation and material is weak, and the radiation propagates in a transparent way. The numerical method in this regime can be well developed by tracking the particle streaming transport. However, for a high opacity (background) material, there is severe interaction between radiation and material with a diminishing photon mean free path. As a result, the diffusive radiative behavior will emerge. In order to solve the kinetic scale based radiative transfer equations numerically, a straightforward way is to use a spatial mesh size which is comparable with photon's mean-free path, i.e., the so-called optical thin cell, and the transport equation can be discretized directly, such as using upwind approach for photon transport. This kind of method is basically a single scale method, where the numerical resolution down to the mean free path is used everywhere in the computation. Most Monte Carlo methods for transport equations belong to this category as well. In this kind of methods, to take such a small cell size will be associated with huge computational cost in the optical thick regime. In order to use a large cell size in comparison with the mean free path in the optical thick region, instead of decoupling the particle transport and collision in the numerical discretization, the coupled transport and collision has to be taken into account in the design of the scheme.

One of the idealized multiscale methods is to develop the so-called asymptotic preserving (AP) scheme for the kinetic equation. When holding the mesh size and time step fixed and as the Knudsen number going to zero, the AP scheme should automatically recover the discrete diffusion solution. AP schemes were first studied in the numerical solution of steady neutron transport problems by Larsen, Morel and Miller [17], Larsen and Morel [16], and then by Jin and Levermore [10,11], and the others. For unsteady problems, one of the AP schemes was constructed based on a decomposition of the distribution function between an equilibrium part and its non-equilibrium derivation, see Klar [13,14], and Jin, Pareschi and Toscani [12] for details. The development of an AP-type discrete ordinate method (DOM) for the multi-frequency radiative transfer equation coupled with material energy equation is an extremely difficult numerical problem [7,8,21], where most well-validated approaches are the Monte Carlo methods.

The UGKS is one of the AP schemes for the transport equations [4,9,24,26]. It not only recovers accurate limiting solutions, such as ballistic transport and diffusion propagation, but also presents reliable solution in the whole transition regime. In UGKS, the mesh size is used directly as a modeling scale for identifying transport dynamics. When the mesh size is on the order of mean free path, the kinetic transport mechanism, such as the modeling process of the Boltzmann equation, is recovered in the numerical evolution [25]. When the mesh size is much larger than the mean free path, the hydrodynamic scale physics, such as the Navier–Stokes (NS) solutions for the flow system and the diffusion equation for the radiative transfer, is obtained. Between these two limits, a smooth transition is constructed and used for the capturing of non-equilibrium phenomena. In UGKS the mesh size and time step are dynamic variables in the evolution model. It may not be difficult to accept this kind of concept if we can realize that all fluid dynamic equations, such as the Boltzmann equation and the NS equations, are constructed based on their specific modeling scales with the corresponding dynamics. More or less the UGKS is using the mesh size and time step to model the dynamics and get the solutions without going through the partial differential equations step [24].

In this paper, an AP UGKS (AP-UGKS) will be developed for the frequency-dependent radiative equations, which are composed of radiation transport and material energy equation. In terms of dynamic modeling, the frequency-dependent radiation system is much more complicated. Within the same spatial cell, both optical thin and thick dynamics can appear for different frequency photons. Not only for the capturing of limiting solutions, accurate solution in the whole transition regime is required in order to capture the dynamics of a continuum spectrum in the frequency domain. The critical ingredient of UGKS is the use of an un-splitting treatment of photon's transport and collision and its automatic recovery of optical thin and thick transport mechanism. The importance of the coupling of transport and collision has been emphasized in a recent paper for the development of AP schemes [5]. The basic steps of AP-UGKS are the following. The multi-group method is first used to discretize the frequency variable, and the discrete-ordinate method (DOM) is employed to discretize the angular distribution of photon's movement. A time evolution integral solution of radiation intensity at different frequency is