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A convective-like energy-stable open boundary condition for simulations of incompressible flows

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A B S T R A C T

We present a new energy-stable open boundary condition, and an associated numerical algorithm, for simulating incompressible flows with outflow/open boundaries. This open boundary condition ensures the energy stability of the system, even when strong vortices or backflows occur at the outflow boundary. Under certain situations it can be reduced to a form that can be analogized to the usual convective boundary condition. One prominent feature of this boundary condition is that it provides a control over the velocity on the outflow/open boundary. This is not available with the other energy-stable open boundary conditions from previous works. Our numerical algorithm treats the proposed open boundary condition based on a rotational velocity-correction type strategy. It gives rise to a Robin-type condition for the discrete pressure and a Robin-type condition for the discrete velocity on the outflow/open boundary, respectively at the pressure and the velocity sub-steps. We present extensive numerical experiments on a canonical wake flow and a jet flow in open domain to test the effectiveness and performance of the method developed herein. Simulation results are compared with the experimental data as well as with other previous simulations to demonstrate the accuracy of the current method. Long-time simulations are performed for a range of Reynolds numbers, at which strong vortices and backflows occur at the outflow/open boundaries. The results show that our method is effective in overcoming the backflow instability, and that it allows for the vortices to discharge from the domain in a fairly natural fashion even at high Reynolds numbers.

1. Introduction

The current work focuses on the outflow/open boundary in incompressible flow simulations and the issue of backflow instability, which refers to the commonly-encountered numerical instability associated with strong vortices or backflows at the outflow or open boundaries. Extending our efforts on this problem [14,12,18], we strive to develop effective and efficient techniques to overcome the backflow instability.

A large class of flow problems involve physically-unbounded domains, such as jets, wakes, boundary layers, and other spatially-developing flows. When numerically simulating such problems, one will need to truncate the domain artificially to a finite size and impose some open (or outflow) boundary condition (OBC) on the artificial boundary. Open boundary conditions are among the most difficult and least understood issues in incompressible flow simulations [28,63], and have commanded a sustained interest of the community for decades. Among the large volume of works accumulated so far on this problem, the traction-free condition [67,25,19,45,3,63,30,46] and the convective (or radiation) boundary condition

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[65,56,28,42,55,22,62,8] are two of the more commonly used. We refer the reader to [63] for a review of this field up to the mid-1990s, and to [57,37,38,34,23,33,54,29,60] for a number of other methods developed by different researchers.

Backflow instability is a commonly encountered issue with outflows or open boundaries at moderate and high Reynolds numbers. Simulations have been observed to instantly blow up when strong vortices or backflows occur at the outflow/open boundary [13,15,70,26]. As pointed out in [18], a certain amount of backflow at the outflow boundary appears harmless at low Reynolds numbers, but when the Reynolds number increases beyond some moderate value, typically several hundred to a thousand depending on the geometry (e.g. in the range $Re = 300$–400 for the flow past a square cylinder in two dimensions), this instability becomes a severe issue to numerical simulations. Commonly-used tricks for flow simulations such as increasing the grid resolution or decreasing the time step size are observed to not help with this instability [15,18].

For certain flow problems (e.g. bluff-body wakes) one way to circumvent this difficulty is to employ a large computational domain and to place the outflow/open boundary far downstream. The idea is to allow for the vortices generated upstream to sufficiently dissipate before reaching the outflow boundary. This is feasible and computationally manageable at moderate Reynolds numbers. But this strategy does not scale with the Reynolds number [14,18], because the domain size essential for numerical stability grows with increasing Reynolds number. As the Reynolds number becomes large, the needed domain size for stability can become very substantial. For example, in the three-dimensional direct numerical simulation of the flow past a circular cylinder at Reynolds number $Re = 10\,000$ [15], a domain size with a wake region 50 times the cylinder diameter in length has been used. Such a large wake region is essential for numerical stability for that Reynolds number, even though the far wake (beyond about 10 times the cylinder diameter) is of little or no physical interest and the meshes/computations in that far region are essentially wasted.

A far more attractive approach is to devise effective open/outflow boundary conditions to overcome the backflow instability. Several such boundary conditions have been studied in the literature. By considering the weak form of the Navier–Stokes equation and symmetrization of the nonlinear term, Bruneau and Fabrie [6,7] proposed to modify the traction condition by a term $\frac{1}{2}(\mathbf{n}\cdot\mathbf{u})^{-}\mathbf{u}$, where $\mathbf{u}$ and $\mathbf{n}$ are respectively the velocity and the outward-pointing unit vector normal to the outflow boundary, and $(\mathbf{n}\cdot\mathbf{u})^{-}$ is defined as $\mathbf{n}\cdot\mathbf{u}$ if $\mathbf{n}\cdot\mathbf{u} < 0$ and is set to zero otherwise. We refer to e.g. [44,20] for applications of this boundary condition in later works. A traction condition containing a term $(\mathbf{n}\cdot\mathbf{u})^{-}\mathbf{u}$, which is very similar to that of [6,7] but without the $\frac{1}{2}$ factor, has been employed in [1,50,59,27,36]. Note that a form $\beta(\mathbf{n}\cdot\mathbf{u})^{-}\mathbf{u}$ where $0 < \beta < 1$ has also been considered in [50]. By considering the energy balance of the system, we have proposed in [14] a boundary condition involving a term $\frac{1}{2}|\mathbf{u}|^2\mathbf{n}\Theta_0(\mathbf{n},\mathbf{u})$, where $|\mathbf{u}|$ is the velocity magnitude and $\Theta_0(\mathbf{n},\mathbf{u})$ is a smoothed step function about $\mathbf{n}\cdot\mathbf{u}$ (see Section 2 for definition). While the role of the term $\Theta_0(\mathbf{n},\mathbf{u})$ can be compared to that of $(\mathbf{n}\cdot\mathbf{u})^{-}$ discussed above, the form $\frac{1}{2}|\mathbf{u}|^2\mathbf{n}$ of the OBC in [14] is very different from those involving $(\mathbf{n}\cdot\mathbf{u})\mathbf{u}$ of the previous works [6,7,1,50,59,27,36]. Another boundary condition developed in [4] employs a penalization of the tangential velocity derivative to allow for improved energy balance. Very recently we have proposed in [18] a family of open boundary conditions, having the characteristic that they all ensure the energy stability of the system even in situations where strong vortices of backflows occur at the outflow/open boundary. This family of boundary conditions contains those of [6,1,27,36,14] as particular cases, and more importantly provides other new forms of open boundary conditions. Several of those forms have been investigated in relative detail in [18].

It is observed that, while some of the above open boundary conditions have existed in the literature for some time, their adoption in production flow simulations appears still quite limited. This is perhaps in part owing to the challenge associated with the numerical implementation of these boundary conditions. All the aforementioned boundary conditions for tackling the backflow instability couple together the velocity and the pressure, and it is not immediately clear how to implement them in numerical simulations. This seems to be exacerbated by the fact that, when these boundary conditions are originally proposed, for most of them their numerical treatments are not discussed or not adequately discussed, especially in the context of the commonly-used splitting or fractional-step type schemes for incompressible flow simulations. It is noted that in the more recent works [14,18] two splitting-type schemes, respectively based on a velocity-correction type strategy [14] and a pressure-correction type strategy [18], are presented to deal with the energy-stable open boundary conditions developed therein. These algorithms de-couple the computations for the pressure and the velocity in the presence of open/outflow boundaries.

The objective of the current paper is twofold. First, we present a new energy-stable open boundary condition that is effective in overcoming the backflow instability for incompressible flow simulations. This boundary condition involves an inertia (velocity time-derivative) term, and can be shown to ensure the energy stability of the system even in the presence of backflows or vortices at the open/outflow boundary. It does not belong to the family of open boundary conditions discussed in [18]. If no backflow occurs at the outflow boundary, this boundary condition can be reduced to a form that can be analogized to the usual convective boundary condition. Hence, we refer to it as the convective-like energy-stable open boundary condition. The current open boundary condition has a prominent feature: it provides a control over the velocity at the open/outflow boundary. In contrast, the family of energy-stable open boundary conditions from [18] and the other aforementioned boundary conditions to address the backflow instability do not provide any control over the velocity at the open/outflow boundary. Therefore, as the vortices pass through the outflow/open boundary, the current boundary condition can lead to smoother velocity patterns in regions at or near the outflow boundary when compared to that of [18].

Second, we present an efficient numerical algorithm for treating the proposed open boundary condition. Our algorithm overall is based on a rotational velocity-correction type splitting approach, and the key issue lies in the numerical treatment of the inertia term in the open boundary condition. At the pressure sub-step our scheme leads to a Robin-type condition for