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Minimizing dispersive errors in smoothed particle magnetohydrodynamics for strongly magnetized medium

Kazunari Iwasaki

Department of Environmental Systems Science, Doshisha University, Tatara Miyakodani 1-3, Kyotanabe City, Kyoto 610-0394, Japan

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ABSTRACT

In this study, we investigate the dispersive properties of smoothed particle magnetohydrodynamics (SPM) in a strongly magnetized medium by using linear analysis. In modern SPM, a correction term proportional to the divergence of the magnetic fields is subtracted from the equation of motion to avoid a numerical instability arising in a strongly magnetized medium. From the linear analysis, it is found that SPM with the correction term suffer from significant dispersive errors, especially for slow waves propagating along magnetic fields. The phase velocity for all wave numbers is significantly larger than the exact solution and has a peak at a finite wavenumber. These excessively large dispersive errors occur because magnetic fields contribute an unphysical repulsive force along magnetic fields. The dispersive errors cannot be reduced, even with a larger smoothing length and smoother kernel functions such as the Gaussian or quintic spline kernels. We perform the linear analysis for this problem and find that the dispersive errors can be removed completely while keeping SPM stable if the correction term is reduced by half. These findings are confirmed by several simple numerical experiments.

1. Introduction

Smoothed particle hydrodynamics (SPH) is an entirely Lagrangian particle method for simulating fluid flows [1,2]. This Lagrangian nature offers major advantages when SPH is applied to problems with a large, dynamic range of spatial scales. Furthermore, SPH can easily incorporate other physics such as self-gravity, radiative transfer, or chemistry. Thus, SPH is widely used in a variety of astrophysical problems such as formation of large-scale structures, galaxies, stars, and planets.

Recently, several authors have tried to extend SPH to magnetohydrodynamics (MHD) because magnetic fields play an important role in a variety of astrophysical environments. In this study, we call SPH for MHD "smoothed particle magnetohydrodynamics" (SPM). Price and Monaghan [3] have developed an SPM method with artificial viscosity and resistivity (also see [4]). Iwasaki and Inutsuka [5] have applied Godunov's method to SPM. We call it "GSPM". Recently, Iwasaki and Inutsuka [6] have modified their original GSPM formulation, based on their derivation of the equation of motion in GSPM from an action principle.

Unfortunately, conservative formulations of SPM are known to inevitably suffer from numerical instability for low β plasma because of negative stress, where β is the ratio of the gas pressure to the magnetic pressure. This instability has already been pointed out by Phillips and Monaghan [7] (also see [8]). Among several methods proposed for removing the numerical instability [7,8,3], an approach by Børve, Omang, and Trulsen [9] is widely used in modern SPM methods (e.g.,

E-mail address: kiwasaki@mail.doshisha.ac.jp.

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[4–6,10]). A broad discussion of stable SPM schemes is found in a review by Price [11]. In the approach of Børve, Omang, and Trulsen [9], a correction term, $\mathbf{B}(\nabla\cdot\mathbf{B})/4\pi$, is subtracted from the right-hand side of the equation of motion. The correction term is essentially zero if $\nabla\cdot\mathbf{B}=0$ is satisfied. By a linear analysis of SPM, Børve, Omang, and Trulsen [12] (hereafter BOT04) found that half of the correction term, or $\mathbf{B}(\nabla\cdot\mathbf{B})/8\pi$, is large enough to remove the numerical instability. This was confirmed by Barnes, Kawata, and Wu [13], who found that half the correction term provided the least error and minimized the violation of energy and momentum conservation in a variety of test calculations. Børve, Omang, and Trulsen [14] have proposed a sophisticated method, wherein the size of the correction term varies among the SPH particles.

However, it is still unclear how the correction term affects the capability of SPM to accurately model fluid flows. Guiding the optimal selection of the amount of correction in a rigorous manner is important. Balsara [15] has investigated the linear stability of various SPH formulations, kernel functions, and ratios of smoothing length to interparticle distance (also see [8,16]). They have suggested an optimal range of parameters from obtained dispersion relations. Although these analyses are valid only for the linear regime and a regular particle configuration, they provide us with a lot of knowledge for achieving further improvements to SPM schemes. Pioneering work for SPM has been done by BOT04, who investigate the dispersive properties of SPM. They parameterize the size of the correction term with a free parameter ξ $(0\le\xi\le 1)$, and use $\xi\times\mathbf{B}\nabla\cdot\mathbf{B}/4\pi$ as the correction term. They found that as mentioned above, $\xi=1/2$ is large enough to stabilize SPM for low β plasma, and also suggested that smoother kernels, such as Gaussian or the quintic spline kernels, reproduce correct phase velocities, while the cubic spline causes large dispersion errors. However, their study is restricted to several linear waves in the long-wavelength limit and to the case with $\xi=1/2$ although many authors still adopt $\xi=1$.

In this study, we investigate detailed dispersive properties of SPM for low β plasma by changing ξ, the kernel functions, and the ratio of smoothing length to interparticle distance. From the results, a suggestion of an optimal choice of the size of the correction term is provided.

The paper is organized as follows: in Section 2, the basic equations of SPM are reviewed. In Section 3, a dispersion relation is derived from the basic equations of SPM and its asymptotic behavior in the long- and short-wavelength limits is discussed. The results of the linear analysis are presented in Section 4. To confirm the results of the linear analysis, several numerical experiments are demonstrated in Section 5. Our results are discussed in Section 6. Section 7 offers a summary.

2. SPM equations

The basic equations of MHD are given by

$$\frac{\partial\rho}{\partial t}+\nabla\cdot(\rho\mathbf{v})=0, \tag{1}$$

$$\frac{\partial v^{\mu}}{\partial t}=\frac{1}{\rho}\nabla^{\nu}T^{\mu\nu}-\frac{\xi}{4\pi\rho}B^{\mu}\nabla^{\nu}B^{\nu}, \tag{2}$$

and

$$\frac{d}{dt}\left(\frac{B^{\mu}}{\rho}\right)=\frac{B^{\nu}}{\rho}\nabla^{\nu}v^{\mu}, \tag{3}$$

where $T^{\mu\nu}$ is the stress tensor,

$$T^{\mu\nu}=-\left(P+\frac{\mathbf{B}^2}{8\pi}\right)\delta^{\mu\nu}+\frac{B^{\mu}B^{\nu}}{4\pi}, \tag{4}$$

and $d/dt=\partial/\partial t+\mathbf{v}\cdot\nabla$ is the Lagrangian time derivative. The second term on the right-hand side of equation (2) is the correction term introduced to remove the numerical instability (BOT04). The parameter ξ specifies the size of the correction term. In this study, ξ is assumed to be constant for all particles. For simplicity, instead of the energy equation, the isothermal equation of state is assumed:

$$P=c^2\rho, \tag{5}$$

where c is the sound speed. In the adiabatic case, the dispersive properties of SPM are expected to be qualitatively the same as those in the isothermal case.

In SPH, the density of the a-th particle is evaluated by the following equation:

$$\rho_a=\sum_b m_b W(\mathbf{x}_a-\mathbf{x}_b,h), \tag{6}$$

where the subscripts denote the particle label, m_b is the mass of the b-th particle, $W(\mathbf{x},h)$ is a kernel function, and h is the smoothing length. In the linear analysis presented in this study, the smoothing length is assumed to be constant. In the numerical experiments shown in Section 5, a variable smoothing length is used.

There are several conservative formulations of SPM. Here, we show two schemes: the standard SPM formulation, the GSPM formulation. The basic equations of standard SPM [9,3,4] are given by