Journal of Computational Physics 302 (2015) 405–419

Contents lists available at ScienceDirect

Journal of Computational Physics

www.elsevier.com/locate/jcp

Efficient finite element method for grating profile reconstruction

Ruming Zhang [*,1], Jiguang Sun [2]

Department of Mathematical Sciences, Michigan Technological University, Houghton, MI 49931, USA

ARTICLE INFO

Article history:
Received 24 November 2014
Received in revised form 9 July 2015
Accepted 11 September 2015
Available online 16 September 2015

Keywords:
Inverse problem
Optimization method
Quasi-Newton method
Finite element method

ABSTRACT

This paper concerns the reconstruction of grating profiles from scattering data. The inverse problem is formulated as an optimization problem with a regularization term. We devise an efficient finite element method (FEM) and employ a quasi-Newton method to solve it. For the direct problems, the FEM stiff and mass matrices are assembled once at the beginning of the numerical procedure. Then only minor changes are made to the mass matrix at each iteration, which significantly saves the computation cost. Numerical examples show that the method is effective and robust.

1. Introduction

Direct and inverse scattering by periodic structures has many applications in diffractive optics, where periodic structures are also known as diffraction gratings [17]. In this paper, we consider the inverse problem of reconstructing grating profiles.

We start with the scattering of time-harmonic electromagnetic plane waves (TE mode) by a penetrable grating. Suppose the grating surface Γ is defined by

$$\Gamma := \left\{ \left(x, f(x)\right) | f(x) \in X \right\},$$

where X is the set of piecewise continuous L-periodic functions in $\mathbb{R}$ (see Fig. 1). The whole space is separated into two parts:

$$\Omega_+ := \{(x, y) \in \mathbb{R}^2 : y > f(x)\}, \quad \Omega_- := \{(x, y) \in \mathbb{R}^2 : y < f(x)\}.$$

The refractive index $n = n_1$ in Ω_+ and $n = n_2$ in Ω_-, where n_1 and n_2 are different constants. For simplicity, we set $n_1 = 1$ throughout the paper. Furthermore, we assume that $\mathrm{Re}\, n_2 \geqslant 0$ and $\mathrm{Im}\, n_2 \geqslant 0$.

Let $k_j = k\sqrt{n_j}$, $j = 1, 2$, where k is the wavenumber. Let $\theta \in (-\pi/2, \pi/2)$ be the incident angle and $\alpha = k_1 \sin\theta$, $\beta^{(1)} = k_1 \cos\theta$. The incident plane wave is given by

$$u^i(x, y) = \exp(i\alpha x - i\beta^{(1)} y).$$

Since $u^i(\cdot, y)$ is α-quasi-periodic, i.e.,

* Corresponding author.
E-mail addresses: rumingz@mtu.edu (R. Zhang), jiguangs@mtu.edu (J. Sun).

[1] The work of R.Z. is supported in part by NSF DMS-1016092/1321391.
[2] The work of J.S. is supported in part by NSF CNIC-1427665.

http://dx.doi.org/10.1016/j.jcp.2015.09.016
0021-9991/

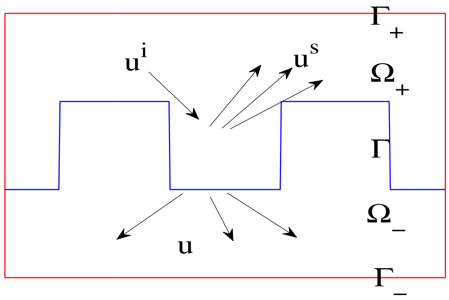

Fig. 1. The physical configuration of the scattering problem.

$$u^i(x+L,y)=\exp(i\alpha L)u^i(x,y),$$

the total field $u=u^i+u^s$ is also α-quasi-periodic,

$$u(x+L,y)=\exp(i\alpha L)u(x,y), \tag{1.1}$$

and satisfies the Helmholtz equation

$$\Delta u+k^2nu=0\,, \quad (x,y)\in\mathbb{R}^2. \tag{1.2}$$

Here, the scattered field u^s in Ω_+ and transmitted field u in Ω_- satisfy the Rayleigh expansion conditions (see [17])

$$u^s(x,y)=\sum_{n\in\mathbb{Z}}u_n^+\exp(i\alpha_n x+i\beta_n^{(1)}y)\,,\quad y>\max_{t\in\mathbb{R}}f(t), \tag{1.3}$$

$$u(x,y)=\sum_{n\in\mathbb{Z}}u_n^-\exp(i\alpha_n x-i\beta_n^{(2)}y)\,,\quad y<\min_{t\in\mathbb{R}}f(t), \tag{1.4}$$

where

$$\alpha_n=\alpha+\frac{2n\pi}{L},\quad \beta_n^{(j)}=\begin{cases}\sqrt{k_j^2-\alpha_n^2}\,, & |\alpha_n|\leqslant k_j,\\ i\sqrt{\alpha_n^2-k_j^2}\,, & |\alpha_n|>k_j.\end{cases}$$

Let Γ_+ and Γ_- be straight lines above and below Γ, respectively, such that

$$\Gamma_+=\{(x,h_+):h_+>\max_{t\in\mathbb{R}}f(t)\},\quad \Gamma_-=\{(x,h_-):h_-<\min_{t\in\mathbb{R}}f(t)\},$$

where $h_+,h_-\in\mathbb{R}$. Let $D=[0,L]\times[h_-,h_+]$. For $\phi^\pm(x,h_\pm)=\sum_{n\in\mathbb{Z}}\hat{\phi}_n^\pm e^{i\alpha_n x}$, the Dirichlet to Neumann maps $T^\pm$ on $\Gamma_\pm$ are given by

$$T^+(\phi^+)=\sum_{n\in\mathbb{Z}}i\beta_n^{(1)}\hat{\phi}_n^+e^{i\alpha_n x}\quad \text{on }\Gamma_+,$$

$$T^-(\phi^-)=\sum_{n\in\mathbb{Z}}i\beta_n^{(2)}\hat{\phi}_n^-e^{i\alpha_n x}\quad \text{on }\Gamma_-.$$

The Rayleigh expansions (1.3)–(1.4) are equivalent to the following boundary conditions

$$\frac{\partial u^s}{\partial y}=T^+u^s\quad \text{on }\Gamma_+, \tag{1.5}$$

$$\frac{\partial u}{\partial y}=-T^-u\quad \text{on }\Gamma_-. \tag{1.6}$$

In this paper, we consider the inverse problem to reconstruct the penetrable grating profile from multiple frequency scattering data. In particular, for incident waves u^i, assuming the scattered fields are measured on Γ_+, i.e., $u^{meas}(x):=u^s(x,h_+)$, we seek a profile $\tilde{\Gamma}$ such that, the scattering data $\tilde{u}^s(x,h_+)$ corresponding to $\tilde{\Gamma}$ coincides (approximately) with the measured data.

Inverse diffractive grating problem has been an active research area. Early uniqueness results include [13], in which it is shown that a perfectly reflecting periodic profile is uniquely determined by the scattered waves due to various incident waves, and [2], where the author proved that the grating profile, which is the interface of a dielectric region with absorption and a perfectly reflecting region, can be uniquely determined by the scattered field on a straight line using a single incident wave. Numerical methods to reconstruct grating profiles have been developed by many researchers. For example, in [6,7], the authors gave a two-step optimization method based on Tikhonov regularization for Dirichlet and transmission problems.