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Development of a Godunov method for Maxwell's equations with Adaptive Mesh Refinement

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ABSTRACT

In this paper we present a second order 3D method for Maxwell's equations based on a Godunov scheme with Adaptive Mesh Refinement (AMR). In order to achieve it, we apply a limiter which better preserves extrema and boundary conditions based on a characteristic fields decomposition. Despite being more complex, simplifications in the boundary conditions make the resulting method competitive in computer time consumption and accuracy compared to FDTD. AMR allows us to simulate systems with a sharp step in material properties with negligible rebounds and also large domains with accuracy in small wavelengths.

1. Introduction

Solving Maxwell's equations in time domain has demonstrated to be a mandatory step in a large number of applications such as Particle In Cell codes [1], laser propagation [2] or antennas simulation [3]. Since analytic solutions are not available for many problems, different schemes of numerical methods have been developed in order to solve Maxwell's equations in time domain.

The two main schemes which have been used are the solution of Time Domain based on Finite Difference (FDTD) or Finite Volumes (FVTD) [4,5]. FDTD is based on Yee's grid [6], where fields are located in staggered space and time grids in order to achieve second order accuracy with a simple discretization. Some studies regarding its stability [7] or modifications of the method to increase its performance [8] have been published since 1966. Studies have also been done regarding the physical boundary conditions while simulating unbounded domains, a known problem with Maxwell solvers. Berenguer designed the Perfectly Matched Layer (PML) [9,10] as an absorbing layer to avoid spurious reflections. FDTD's success is based on its simplicity and ability to handle complex geometries, but some drawbacks are to be found like the difficulties at representing boundaries between regions of the domain with different electromagnetic properties or the delay of numerical wave speed introduced by numerical dispersion [4,5,11]. Regarding FVTD, schemes in different grids have been presented such as [12] for Cartesian grids or, more recently [11] for unstructured meshes.

While trying to simulate problems with different scales or big domains compared to the smallest wavelength of the electromagnetic field, computational limits make us look for methods with local refinement in the grid such as local mesh refinement [13] or Adaptive Mesh Refinement (AMR). AMR [14] is a simulation technique which consists in calculating the solution on different levels with more resolution in each, both in time and space domains. AMR uses high resolution grids where they are needed in order to properly represent waves with small wavelength compared to the base cell width.

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Another case where AMR uses high resolution is while dealing with steps in materials or sources. AMR uses low resolution for the rest of the domain, avoiding heavy and useless calculations.

Our first attempt to apply AMR to solve Maxwell's equations was using the FDTD scheme. Due to the *infinite memory* [1] of Maxwell's equations, any numerical error in the interpolation at boundaries between AMR grids acts as a source of electromagnetic field. We have tried several interpolation methods, but FDTD is a scheme with low numerical diffusion, so the artificial sources at the boundaries between different AMR grids remain for the whole simulation's time, destroying the accuracy of the method. The FDTD and AMR technique have been studied from different points of view. Costas [15] proposed in 2005 a simple scheme with linear interpolation, and J.L. Vay et al. [16] have more recently published a far more complex scheme using extra grids in order to avoid production of spurious sources.

Our final approach involves considering Maxwell's equations in their conservative form and solving them in a similar way as done in Computational Fluid Dynamics (CFD), with a Godunov method such as the Piecewise Parabolic Method (PPM) [17]. We have used a second order in time and fourth in space scheme which gives us a global second order of accuracy for the whole scheme, but other might had been used in order to increase accuracy [19]. The use of PPM with AMR allows us to fully simulate big domains with coarse resolution while waves and interactions are followed by the resolved mesh. The proper choice of the limiters [18,19] also allows us to simulate discontinuities in material electromagnetic properties as will be seen in Section 6. Finally, a new absorbing layer for the physical boundary has been developed based on characteristic fields decomposition which have a good absorption rate while consuming less computational resources than Berenguer's PML.

For the sake of simplicity in Section 2 we describe the numerical method we are using in 1D and also the physical boundary conditions for unbound domains that we have developed taking advantage of our Godunov method properties. Section 3 describes how AMR interaction between levels is managed and results for the main test of a plane wave crossing through a steady AMR box are presented. Then, the entire Section 4 is devoted to the 3D Riemann solver. In Section 5 we explain how our boundary conditions work in 3D. Finally, some results will be delivered in Section 6 showing that this method produces good enough results compared to FDTD.

2. The Godunov method

2.1. Maxwell's equations in their conservative form

The system of equations to be solved is the 3D set of rotational Maxwell's equations (1).

$$\left.\begin{aligned}\frac{\partial \mathbf{B}}{\partial t} &= -\nabla\times\mathbf{E}\\ \frac{\partial \mathbf{D}}{\partial t} &= \nabla\times\mathbf{H}-\mathbf{J}\end{aligned}\right\} \tag{1}$$

We will consider here only **B** and **D** fields for the calculations. In order to avoid **E** and **H** fields, constitutive equations with fixed in time, diagonal ϵ and μ tensors will be used as in Eq. (2).

$$\begin{aligned}\mathbf{D} &= \epsilon\mathbf{E}\\ \mathbf{B} &= \mu\mathbf{H}\end{aligned} \tag{2}$$

The equation for the conservative laws is:

$$\frac{\partial \mathbf{u}}{\partial t}+\nabla\mathbf{F}(\mathbf{u}) = \frac{\partial \mathbf{u}}{\partial t}+\frac{\partial \mathbf{F}_x(\mathbf{u})}{\partial x}+\frac{\partial \mathbf{F}_y(\mathbf{u})}{\partial y}+\frac{\partial \mathbf{F}_z(\mathbf{u})}{\partial z} = \mathbf{S} \tag{3}$$

The conservative approach for the problem consists in writing Eq. (1) in conservative form (3). In 3D this is done by using variables and fluxes in Eq. (4).

$$\mathbf{u}=\begin{bmatrix} D_x\\ D_y\\ D_z\\ B_x\\ B_y\\ B_z\end{bmatrix};\quad \mathbf{F}_x=\begin{bmatrix}0\\ \frac{B_z}{\mu_z}\\ -\frac{B_y}{\mu_y}\\ 0\\ -\frac{D_z}{\epsilon_z}\\ \frac{D_y}{\epsilon_y}\end{bmatrix};\quad \mathbf{F}_y=\begin{bmatrix}-\frac{B_z}{\mu_z}\\ 0\\ \frac{B_x}{\mu_x}\\ \frac{D_z}{\epsilon_z}\\ 0\\ -\frac{D_x}{\epsilon_x}\end{bmatrix};\quad \mathbf{F}_z=\begin{bmatrix}\frac{B_y}{\mu_y}\\ -\frac{B_x}{\mu_x}\\ 0\\ -\frac{D_y}{\epsilon_y}\\ \frac{D_x}{\epsilon_x}\\ 0\end{bmatrix};\quad \mathbf{S}=\begin{bmatrix}J_x\\ J_y\\ J_z\\ 0\\ 0\\ 0\end{bmatrix} \tag{4}$$

Eigenvalues in direction γ are:

$$\lambda_0 = 0\,(\text{double})\,;\qquad \lambda_{\alpha\beta} = c_{\alpha\beta} = \pm\frac{1}{\sqrt{\epsilon_\alpha\mu_\beta}};\qquad \lambda_{\beta\alpha} = c_{\beta\alpha} = \pm\frac{1}{\sqrt{\epsilon_\beta\mu_\alpha}} \tag{5}$$

for $\alpha, \beta, \gamma =$ any permutation of x, y, z.