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Well-balanced central-upwind scheme for a fully coupled shallow water system modeling flows over erodible bed

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ABSTRACT

Intense sediment transport and rapid bed evolution are frequently observed under highly-energetic flows, and bed erosion sometimes is of the same magnitude as the flow itself. Simultaneous simulation of multiple physical processes requires a fully coupled system to achieve an accurate hydraulic and morphodynamical prediction. In this paper, we develop a high-order well-balanced finite-volume method for a new fully coupled two-dimensional hyperbolic system consisting of the shallow water equations with friction terms coupled with the equations modeling the sediment transport and bed evolution.

The nonequilibrium sediment transport equation is used to predict the sediment concentration variation. Since bed-load, sediment entrainment and deposition have significant effects on the bed evolution, an Exner-based equation is adopted together with the Grass bed-load formula and sediment entrainment and deposition models to calculate the morphological process. The resulting 5×5 hyperbolic system of balance laws is numerically solved using a Godunov-type central-upwind scheme on a triangular grid. A computationally expensive process of finding all of the eigenvalues of the Jacobian matrices is avoided: The upper/lower bounds on the largest/smallest local speeds of propagation are estimated using the Lagrange theorem. A special discretization of the bed-slope term is proposed to guarantee the well-balanced property of the designed scheme. The proposed fully coupled model is verified on a number of numerical experiments.

1. Introduction

Shallow water systems are widely used in predicting hydrodynamics of surface flows such as water flows in rivers, channels, flood plains and coastal regions. It is well-known that the shallow water systems can accurately predict the hydraulic parameters under conditions of slow erosion and low sediment concentration. However, in cases of highly energetic flows like dam-break or flood flows, the effects of intense sediment exchange and rapid bed evolution cannot be neglected.

In recent years, a number of dam-break hydraulics models have been developed to predict the fluid flow and sediment transport [37,44,46,47,59,60]. However, the majority of the current models are based on the decoupled equations of flow and sediment transport and thus they are restricted to erosion rates which are considerably weaker than the flow, see, e.g.,

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[37,56]. However, dam-break flows may induce intense sediment transport and rapid morphological changes which are not independent processes but have mutually significant interactions with the fluid flow. In many dam-break cases and in strong fluvial processes in rivers, the bed evolution is almost of the same order of magnitude as the free-surface level changes [55]. Hence, developing a dam-break model that takes into account coupling hydraulic model with the sediment transport and bed evolution equations is required in such cases.

Many dam-break models based on fixed-bed conditions have been developed, see, e.g., [6–8,44,47,59,60]. In order to evaluate morphological changes during the high-energy dam-break flows, some earlier numerical models use uncoupled strategy to first solve the hydrodynamic model, and then to solve the sediment transport and bed deformation equations separately, see, e.g., [19,20,41,46]. More recently, noting that the flow, sediment transport and bed evolution can be strongly coupled to each other while the rate of bed deformation being considerable compared to that of the flow evolution, several coupled mathematical modeling strategies for simulating dam-break flows over mobile beds have been developed. Fagherazzi and Sun [17] presented a 1-D coupled system consisting of a simplified shallow water system, sediment concentration and bed level calculation. Cao et al. [11] proposed a 1-D coupled model and the sediment entrainment and deposition are considered to update the bed-levels. Wu and Wang presented a similar 1-D coupled model for dam-break flows over erodible beds in [51] including the bed-load transport. Capart and Young [12] developed a one-dimensional (1-D) coupled dam-break model based on an explicit finite-difference algorithm.

Several two-dimensional (2-D) coupled models for dam-break flows over mobile beds have been recently introduced. Simpson and Castelltort [43] presented a 2-D model for the free surface flow, sediment transport and bed level change based on the 1-D model from [11]. Yue et al. [54] extended the 1-D model from [11] to 2-D for modeling alluvial processes with intense sediment transport and rapid bed evolution. Similar 2-D coupled mathematical models have been also studied in [24,52]. Li and Duffy [36] proposed a fully coupled 2-D system including the modified shallow water system, sediment transport and bed evolution. In their study, the bed evolution formula is rewritten to couple the sediment entrainment and deposition effects by substituting the sediment transport equation into it. Meanwhile, the sediment exchange terms are treated as source terms which will not affect the solution of the hyperbolic system. In [5], Benkhaldoun et al. presented a coupled 2-D model, which consists of both bed-load and sediment exchange in the morphological equation, and the bed-load terms are treated as a flux term coupled in the hyperbolic system. Hudson and Sweby [25] investigated the accuracy and determined the validity of both the steady approach and unsteady approach of five different types of governing formulations coupling the morphology continuity equation on rectangular meshes. They concluded that one conservative reformulation form based on the unsteady approach was overall the best. However, the drawback of this form is its dependence of the Jacobian matrix on z, which was shown and discussed in [42]. Castro Díaz et al. [13] studied the numerical approximation of bed-load sediment transport due to shallow layer flows on unstructured meshes with a second-order MUSCL-type reconstruction. Murillo and Garcia-Navarro [39] studied an Exner-based coupled model for 2-D transient flows over erodible beds on triangular unstructured meshes and developed a general "Grass formula" format expressing several commonly used empirical bed-load formulas. A similar scheme is adopted in the current study using the Meyer–Peter and Müller formula. Soares-Frazão and Zech [45] built a coupled system using the HLLC method and proposed an approximate analytical expression for the wave celerities, which was valid for any Froude number.

In this paper, we propose a new fully coupled hyperbolic system consisting of hydrodynamic model, sediment transport and morphological evolution. The obtained hyperbolic system of balance laws consists of five coupled equations, for which no analytical expressions for the eigenstructure are available. This makes it difficult to develop a robust and efficient upwind scheme for the proposed model. We develop a Riemann-problem-solver-free finite volume method for the studied fully coupled system. Our method is based on the efficient, robust and highly accurate semi-discrete central-upwind scheme, originally introduced in [28–30,33] for general multidimensional hyperbolic systems of conservation laws. The central-upwind scheme was first extended to the Saint-Venant system of shallow water equations in [27]. A more robust, well-balanced and positivity preserving central-upwind scheme for the shallow water equations was developed on both Cartesian [31] and triangular [8] grids. These schemes are capable of exactly preserving "lake at rest" steady-state solutions and preserving positivity of the computed water depth. In this paper, we extend the second-order semi-discrete central-upwind scheme to the fully coupled shallow water system modeling flows over erodible bed. Since, according to the central-upwind methodology, no (generalized) Riemann problems are to be (approximately) solved, the numerical fluxes are obtained in a straightforward manner. The one-sided local speeds needed to evaluate numerical fluxes are estimated using the Lagrange theorem [34,38], which significantly reduces the numerical cost (the same approach was used in [14,32] in the context of the two- and three-layer shallow water equations). We complete the development of the well-balanced (in the sense that it is capable of exactly preserving "lake at rest" states) central-upwind scheme by designing a positivity preserving piecewise linear reconstruction and a well-balanced quadrature for the bed-slope terms.

This paper is organized as follows. In Section 2, we present the governing equations, for which in Section 3 we develop the central-upwind scheme. Several numerical experiments are then presented in Section 4. Some concluding remarks complete the paper in Section 5.

2. Governing equations

In this section, we will present the model equations (Section 2.1) and write down the studied model in the vector form (Section 2.2).