Journal of Computational Physics 300 (2015) 269–287

Contents lists available at ScienceDirect

Journal of Computational Physics

www.elsevier.com/locate/jcp

Preventing numerical oscillations in the flux-split based finite difference method for compressible flows with discontinuities

Zhiwei He [a], Yousheng Zhang [a], Xinliang Li [b], Li Li [b], Baolin Tian [a,*]

[a] Laboratory of Computational Physics, Institute of Applied Physics and Computational Mathematics, Beijing 100094, China
[b] State Key Laboratory of High-temperature Gas Dynamics, Institute of Mechanics, Chinese Academy of Sciences, Beijing 100190, China

ARTICLE INFO

Article history:
Received 4 January 2015
Received in revised form 3 July 2015
Accepted 25 July 2015
Available online 30 July 2015

Keywords:
Compressible flows
Contact discontinuity
Material interface
Numerical oscillations
Finite difference method
Flux vector splitting
WENO

ABSTRACT

In simulating compressible flows with contact discontinuities or material interfaces, numerical pressure and velocity oscillations can be induced by point-wise flux vector splitting (FVS) or component-wise nonlinear difference discretization of convection terms. The current analysis showed that the oscillations are due to the incompatibility of the point-wise splitting of eigenvalues in FVS and the inconsistency of component-wise nonlinear difference discretization among equations of mass, momentum, energy, and even fluid composition for multi-material flows. Two practical principles are proposed to prevent these oscillations: (i) convective fluxes must be split by a global FVS, such as the global Lax–Friedrichs FVS, and (ii) consistent discretization between different equations must be guaranteed. The latter, however, is not compatible with component-wise nonlinear difference discretization. Therefore, a consistent discretization method that uses only one set of common weights is proposed for nonlinear weighted essentially non-oscillatory (WENO) schemes. One possible procedure to determine the common weights is presented that provided good results. The analysis and methods stated above are appropriate for both single- (e.g., contact discontinuity) and multi-material (e.g., material interface) discontinuities. For the latter, however, the additional fluid composition equation should be split and discretized consistently for compatibility with the other equations. Numerical tests including several contact discontinuities and multi-material flows confirmed the effectiveness, robustness, and low computation cost of the proposed method.
© 2015 Elsevier Inc. All rights reserved.

1. Introduction

Flows evolving from contact discontinuities or material interfaces are present in a wide range of problems, such as inertial confinement fusion, supernova explosions, high-speed combustion, cavitation bubble clouds, industrial coatings, and fluidized beds. The Richtmyer–Meshkov instability [1] and Rayleigh–Taylor instability [2] are two archetypical examples. Such flows give rise to challenging problems both theoretically and computationally [3].

Much of the theoretical and practical evidence accumulated over many years suggest that, in the presence of shocks, there is no alternative but to use conservative methods [4]. However, anomalies or computational difficulties with using conservative methods have been reported, such as the low Mach number flow [5–10], sonic point glitch [11], carbuncle phenomenon [12], and others [4,13–16]. In this paper, we focus on the problems of compressible flows with contact discon-

* Corresponding author.
E-mail address: tian_baolin@iapcm.ac.cn (B. Tian).

http://dx.doi.org/10.1016/j.jcp.2015.07.049
0021-9991/© 2015 Elsevier Inc. All rights reserved.

tinuities and/or material interfaces. Anomalies are also encountered when these problems are solved by using conservative methods.

First, numerical oscillations can be observed in problems with single-material contact discontinuities. As discussed by Clerc [17], when a conservative Godunov-type finite volume method (FVM) is used for these problems, spurious oscillations tend to be generated. These oscillations slow down the time-marching procedures for steady-state computations and spoil the numerical flow fields. Clerc analyzed the source of these oscillations and found that these oscillations are produced provided that the isobars are not a straight line. Johnsen [18] found that another kind of velocity and pressure oscillations is induced when high-order component-wise weighted essentially non-oscillatory (WENO) reconstruction is used. These oscillations are caused by the inconsistent reconstruction between the mass, momentum, and energy equations. The reconstruction of primitive or characteristic variables must be adopted [18].

Second, some computational difficulties are encountered when solving multi-material flows consisting of pure fluids separated by material interfaces. In these problems, an additional equation quantifying the fluid composition must be introduced to close the governing equations. In early algorithms for computing compressible multi-material flows, the discontinuous nature of the fluid composition was represented by the mass fraction, ratio of specific heats, or level-set function and evolved according to an advection equation coupled to the Euler equations [19]. However, Toro [4] found that, when the fluid composition is represented by the ratio of specific heats, pressure oscillations are introduced even by using various classical first-order schemes. Abgrall and Karni [3] found that, no matter what is used to represent the fluid composition, velocity and pressure oscillations already appear in the first-order computations and are not removed by going to a higher order. Any Godunov-type scheme that is fully conservative cannot maintain pressure equilibrium and will develop a pressure oscillation across material interfaces [20].

In order to overcome these difficulties, some fully nonconservative [21] and non-strictly conservative [20,22–25] approaches have been proposed. Among these attempts, the quasi-conservative approach proposed by Abgrall [20] seems promising and has been extended to problems with complicated equations of state [26–28]. The main idea of this approach is that the proper variable to represent the fluid composition is a function of the ratio of specific heats (i.e., $\Gamma = \frac{1}{\gamma-1}$, where γ is the ratio of specific heats) and the equation of Γ cannot be discretized independently of the discretization of the conserved variables in the Euler equations. This approach has been realized with the low-order finite volume method (FVM), such as first- and second-order variable reconstruction with various Riemann solvers [29]. Recently, this approach was extended to high-order WENO reconstruction with the HLLC solver [30] by Johnsen et al. [19,31]. In contrast to the low-order schemes, they found that new numerical oscillations are produced when high-order nonlinear WENO schemes are used. They analyzed the numerical oscillations and suggested that only the reconstruction of primitive variables can eliminate the spurious oscillations because the specific heat ratio is not constant [31]. Nonomura et al. [32] more comprehensively discussed the numerical oscillations of compressible multi-material flows and concluded that primitive variable interpolation can effectively eliminate numerical oscillations and that a better choice of fully conservative or quasi-conservative forms depends on the problem. Here, we emphasize that Nonomura et al. selected the weighted–compact-nonlinear-scheme (WCNS) variable interpolation finite difference formulation [33] to take over the numerical technique developed with the FVM [19].

On the other hand, for most Richtmyer–Meshkov instability and Rayleigh–Taylor instability problems, the initial configurations are unsteady, and the flows eventually evolve to turbulence. This remains a challenge for theoretical and experimental studies [34]. Instead, numerical simulations have become a powerful tool for these studies, particularly the direct numerical simulation of compressible turbulence. This requires the numerical methods to be high-fidelity in terms of accuracy, resolution, and capturing discontinuities. As a consequence, various high-order methods have been developed. Among these methods, the finite difference method (FDM), especially the high-order component-wise FDM [35], has been welcomed for its simplicity, effectiveness, and low computational cost [36–38].

However, in contrast to the frequent use of the FVM to simulate multi-material flows, the high-order FDM is used more often to simulate single-material flows. So far, there are just a few results computed with the FDM for compressible flows with contact discontinuities and/or material-interfaces. In 2003, Marquina and Mulet [39] directly solved the conservation form of the governing equations with the FDM. In this simulation, the convective fluxes were first split with Marquina's flux splitting method [40], and WENO schemes [41] were then used to obtain the numerical fluxes. Unfortunately, they observed spurious pressure and velocity oscillations at the material interfaces that were assumed to be too small to interfere with the physics of their simulation. Recently, Terashima et al. [42] directly implemented central finite difference schemes to simulate multi-material flows by introducing consistent local artificial diffusion terms to suppress the numerical oscillations of pressure and velocity.

To our knowledge, there has been no general analysis on the computational difficulties of the flux-split based FDM, especially for the frequently used nonlinear finite difference WENO schemes, when solving problems of compressible flows with contact discontinuities and/or material interfaces. As argued by Nonomura et al. [32], the technique proposed by Johnsen and Colonius [19] cannot be applied to the finite difference WENO schemes because these schemes do not include primitive variable reconstruction. In this paper, we present a systematic analysis of this question. The logic and main results are as follows:

- For compressible flows with contact discontinuities or material interfaces, nonphysical oscillations of the pressure and velocity will be produced as long as the implementation of the FDM involves either one of the following operations: (i) point-wise flux vector splitting (FVS) is used to split convective fluxes or (ii) a nonlinear finite difference scheme

Download English Version:

https://daneshyari.com/en/article/518071

Download Persian Version:

https://daneshyari.com/article/518071

Daneshyari.com