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Partitioned coupling strategies for multi-physically coupled radiative heat transfer problems

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ABSTRACT

This article aims to propose new aspects concerning a partitioned solution strategy for multi-physically coupled fields including the physics of thermal radiation. Particularly, we focus on the partitioned treatment of electro–thermo-mechanical problems with an additional fourth thermal radiation field. One of the main goals is to take advantage of the flexibility of the partitioned approach to enable combinations of different simulation software and solvers. Within the frame of this article, we limit ourselves to the case of nonlinear thermoelasticity at finite strains, using temperature-dependent material parameters. For the thermal radiation field, diffuse radiating surfaces and gray participating media are assumed. Moreover, we present a robust and fast partitioned coupling strategy for the fourth field problem. Stability and efficiency of the implicit coupling algorithm are improved drawing on several methods to stabilize and to accelerate the convergence. To conclude and to review the effectiveness and the advantages of the additional thermal radiation field several numerical examples are considered to study the proposed algorithm. In particular we focus on an industrial application, namely the electro–thermo-mechanical modeling of the field-assisted sintering technology.

1. Introduction

As a result of the ever increasing computational capacity, physical processes can be modeled with increased accuracy, allowing more realistic predictions of real-life problems. Here, multi-physically coupled processes leading to complex physical models are of special interest for many industrial applications. Especially the transfer of thermal energy is an important issue – and in many cases an in-depth realistic description needs to take the thermal interactions between the body and its environment into consideration. Thereby, in addition to heat conduction and convection, a radiative heat transfer occurs. As thermal radiation is caused by electromagnetic waves, heat transfer will occur under a vacuum too, meaning that there is no specific medium necessary to transfer thermal energy. Thermal radiation is of particular significance for industrial applications involving highly heated tools, where it may be the dominating mechanism to transfer thermal energy. In this article, the influence of thermal radiation is studied on the electro–thermo-mechanical process of the field-assisted sintering technology [38,39], among others, which is an innovative processing technique for the sintering of powder materials.

In order to simulate a multi-physically coupled process, there are two fundamental strategies to solve this problem. The first one contemplates the entire system simultaneously within one time increment and is denoted as a *monolithic scheme*.

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This scheme is unconditionally stable for implicit time-integration, but involves asymmetrical global system matrices for the full problem, which may lead to extremely large systems.

The *partitioned approach* as the second strategy divides the entire multi-physical system into an iteration of several sub-problems, so that the fields involved are solved individually. Here, it might be necessary to solve the sub-problems successively until the fields are balanced. The main drawback of the partitioned approach stems from its property of being only conditionally stable. To this end, methods to stabilize the algorithm are required. Such methods are known from strongly coupled fluid–structure interactions [6,18,30,32,47] and can also be applied to thermo-mechanical [13] and electro–thermo-mechanical [15] problems. The implementation of these methods into the overall algorithm is straightforward and they work independently of the field solvers. Furthermore, the partitioned approach is much more flexible than the monolithic scheme, so different solvers, software and discretizations techniques may be used for the individual fields. Due to the high flexibility of the partitioned approach, the electro–thermo-mechanical problem, for example, can easily be extended by a thermal radiation field, so that detailed radiative heat transfer effects may be taken into account too.

The responsible transport equation for thermal radiation can be approximated by different numerical methods. In the general case of thermal radiation in a participating medium, the most commonly used methods are the *method of spherical harmonics* [28] and the *discrete ordinates method* in its modern finite volume formulation [7] to name a few. An additional approximation method is the *Monte Carlo method* proposed by [24], which can model irregular radiation behavior. However, due to its statistical nature, it is difficult to combine this method with others.

In a transparent medium or a vacuum, the heat transfer reduces to a purely thermal radiation. The radiation arising from the participating surfaces can be determined by means of *view factors*. Due to their plain structure, we employ the *method of spherical harmonics*, the *finite volume discrete ordinates method* as well as the *view factor method* (VFM) in this article. The benefits of these methods are studied in detail in this article, especially with regard to the integration into the partitioned solution strategy, the accuracy and computational efficiency.

The structure of this paper is as follows: we start with a brief overview of the electro–thermo-mechanical modeling and the required linearization of each individual field in Section 2. Thereafter, the radiative heat transfer and the corresponding numerical methods for vacuum and participating medium are treated in Section 3, followed by the description of the partitioned coupling algorithm in Section 4. Here, we also consider the data transfer between different discretization schemes and methods leading to convergence acceleration. In Section 5, the proposed solution strategy is analyzed based on several numerical examples, followed by concluding remarks in Section 6.

2. Electro–thermo-mechanical modeling

This section provides a brief overview of the partitioned treatment of electro–thermo-mechanically coupled problems. For this purpose, the mechanical, the thermal and the electric fields are treated separately – allowing us to point out the interdependencies between the individual fields. A linearization is required too, as the mechanical and the thermal field are nonlinear. The electric field is considered to be linear in the electric potential, so there is no linearization required here.

2.1. The mechanical, thermal and electric fields

We start with the mechanical field and choose a thermoelastic material that includes large deformations under finite strains. The material body in the three-dimensional Euclidean space is defined at $t = 0$ in the reference configuration Ω_0 as $\mathcal{B}_0$ and at $t > 0$ in the current configuration Ω_t as $\mathcal{B}_t$. The motion of the points between the various configurations, where $\mathbf{X}$ denotes the coordinates of the points in the reference configuration and $\mathbf{x}$ the coordinates of the points in the current configuration, can be described by a nonlinear invertible mapping function $\varphi(\mathbf{X}, t) : \mathcal{B} \rightarrow \mathbb{R}^3$. The displacements between the points are given by $\mathbf{u}(\mathbf{X}, t) = \varphi(\mathbf{X}, t) - \mathbf{X}$. In order to describe the kinematics of the thermoelastically coupled problem, the resulting deformation gradient $\mathbf{F} = \operatorname{Grad}\varphi(\mathbf{X}, t)$ is decomposed by a multiplicative split into a mechanical and a thermal part $\mathbf{F} = \mathbf{F}_\mathrm{M}\mathbf{F}_\Theta$ where the latter one is assumed to be purely volumetric $\mathbf{F}_\Theta = \vartheta(\Theta)\mathbf{I}$. The scalar $\vartheta(\Theta) = \exp[\alpha_\Theta \Delta\Theta]$ describes the thermal stretch ratio where α_Θ is the thermal expansion coefficient and $\Delta\Theta = \Theta - \Theta_0$ the difference in the temperature $\Theta(\mathbf{x}, t)$ relative to the reference temperature Θ_0. Additionally, $\mathrm{J} = \det \mathbf{F} = \mathrm{J}_\mathrm{M}\mathrm{J}_\Theta \geq 0$ is introduced denoting the determinant of the deformation gradient. Further, the mechanical part $\mathbf{F}_\mathrm{M} = \hat{\mathbf{F}}_\mathrm{M}\bar{\mathbf{F}}_\mathrm{M}$ is decomposed into a volume changing part $\hat{\mathbf{F}}_\mathrm{M}$ and a volume preserving part $\bar{\mathbf{F}}_\mathrm{M}$. According to this split, we have

$$
\begin{aligned}
\hat{\mathbf{F}}_\mathrm{M} &= (\det \mathbf{F}_\mathrm{M})^{1/3}\mathbf{I}, \qquad && \mathrm{J}_\mathrm{M} := \det \mathbf{F}_\mathrm{M} = \mathrm{J}/\mathrm{J}_\Theta, \quad \mathrm{J}_\Theta = \det \mathbf{F}_\Theta = \vartheta^3(\Theta)\,, \\
\bar{\mathbf{F}}_\mathrm{M} &= (\det \mathbf{F}_\mathrm{M})^{-1/3}\mathbf{F}_\mathrm{M}, \qquad && \bar{\mathrm{J}}_\mathrm{M} := \det \bar{\mathbf{F}}_\mathrm{M} = 1,
\end{aligned}
\tag{1}
$$

which implies that $\bar{\mathbf{F}}_\mathrm{M} = \bar{\mathbf{F}} = \mathrm{J}^{-\frac{1}{3}}\mathbf{F}$ holds. For details concerning the multiplicative decomposition of the deformation gradients, the interested reader is referred [22,23,34,56] for instance. Moreover, we assume that the electric field has no direct influence on the mechanical field – meaning that electric current flowing through a solid will neither produce any deformation nor cause stresses in the material.

We take advantage of the right Cauchy–Green tensor $\mathbf{C} = \mathbf{F}^\mathrm{T}\mathbf{F}$ and the 2nd Piola–Kirchhoff stress tensor $\mathbf{S} = 2\partial\psi/\partial\mathbf{C}$ with the Helmholtz free-energy $\psi(\mathbf{C}, \Theta)$ to set up the nonlinear boundary value problem of the *mechanical* field, see [22,23] for example. In the reference configuration, the boundary value problem reads