Journal of Computational Physics 300 (2015) 387–422

Contents lists available at ScienceDirect

Journal of Computational Physics

www.elsevier.com/locate/jcp

An upwind vertex centred Finite Volume solver for Lagrangian solid dynamics

Miquel Aguirre [1], Antonio J. Gil *, Javier Bonet, Chun Hean Lee

Zienkiewicz Centre for Computational Engineering, College of Engineering, Swansea University, Singleton Park, SA2 8PP, United Kingdom

ARTICLE INFO

Article history:
Received 18 September 2014
Received in revised form 5 May 2015
Accepted 11 July 2015
Available online 21 July 2015

Keywords:
Fast dynamics
Mie–Grüneisen
Finite Volume method
Riemann solver
Incompressible
Locking
Shock capturing

ABSTRACT

A vertex centred Jameson–Schmidt–Turkel (JST) finite volume algorithm was recently introduced by the authors (Aguirre et al., 2014 [1]) in the context of fast solid isothermal dynamics. The spatial discretisation scheme was constructed upon a Lagrangian two-field mixed (linear momentum and the deformation gradient) formulation presented as a system of conservation laws [2–4]. In this paper, the formulation is further enhanced by introducing a novel upwind vertex centred finite volume algorithm with three key novelties. First, a conservation law for the volume map is incorporated into the existing two-field system to extend the range of applications towards the incompressibility limit (Gil et al., 2014 [5]). Second, the use of a linearised Riemann solver and reconstruction limiters is derived for the stabilisation of the scheme together with an efficient edge-based implementation. Third, the treatment of thermo-mechanical processes through a Mie–Grüneisen equation of state is incorporated in the proposed formulation. For completeness, the study of the eigenvalue structure of the resulting system of conservation laws is carried out to demonstrate hyperbolicity and obtain the correct time step bounds for non-isothermal processes. A series of numerical examples are presented in order to assess the robustness of the proposed methodology. The overall scheme shows excellent behaviour in shock and bending dominated nearly incompressible scenarios without spurious pressure oscillations, yielding second order of convergence for both velocities and stresses.

1. Introduction

In practical engineering applications involving extremely complex geometries, meshing typically represents a large portion of the overall design and analysis time. In the computational mechanics community, the ability to perform calculations on tetrahedral meshes has become increasingly important. For these reasons, the automated tetrahedral mesh generators by means of Delaunay and advancing front techniques [6] have recently received increasing attention in a number of important application areas, such as cardiovascular tissue modelling [7], crash impact simulation [8], blast and fracture mechanics and complex multi-physics problems [9–12].

Unfortunately, modern tetrahedral element technology in solid mechanics (e.g. ANSYS AUTODYN, LS-DYNA, ABAQUS/Explicit, Altair HyperCrash), typically based on the use of the traditional Finite Element based second order displacement formulation [13,14], possesses several distinct disadvantages, namely: (1) Reduced order of convergence for strains and

* Corresponding author.
E-mail address: a.j.gil@swansea.ac.uk (A.J. Gil).
[1] Present address: Vascular Surgery Department, University of Michigan, North Campus Research Complex, 2800 Plymouth Road, Ann Arbor, MI 48109-2800, USA.

http://dx.doi.org/10.1016/j.jcp.2015.07.029
0021-9991/

stresses [15,16]; (2) High frequency noise in the vicinity of shocks [17–20]; (3) Stability issues associated to shear locking, volumetric locking [21] and pressure checkerboard instabilities [22].

To address the shortcomings mentioned above, a wide variety of enhanced discretisation technologies have been developed. As an example, for the case of nearly incompressible materials, the mean dilatational hexahedral formulation [23–25] where constant interpolation is used for the calculation of volumetric stresses [26] has attracted industrial interest, as the modifications associated to the classical displacement based formulation are minor. High order elements [27–29] (also known as p-refinement) can alternatively be used. However, the increase in the number of integration points can drastically reduce the computational efficiency of these schemes in comparison with low order approaches [30], specially when either complex constitutive laws (i.e. anisotropic visco-elastic models are often used in the medical field [31]) or contact surfaces [32], or a combination of both, must be modelled.

The success of nodally integrated tetrahedral elements was first reported in [33], where the volumetric strain energy functional was approximated through averaged nodal pressures. Extensive efforts have since been made to further develop this class of averaged nodal strain technologies with the use of various types of stabilisation [34–39]. Despite exhibiting geometric locking-free behaviours, the resulting formulation still suffers from spurious hydrostatic pressure fluctuations when simulating nearly incompressible materials.

Several authors have presented alternative $\boldsymbol{p}$–$\boldsymbol{F}$ mixed formulations in both Lagrangian solid and gas dynamics [1–4, 40–42]. Specifically, in references [1,3,40,41], the authors presented a mixed conservation law for applications in Lagrangian fast solid dynamics, which are spatially discretised using tailor-made CFD technology. A variant of this approach has been presented in [9,10] in the context of fluid–structure interaction. The use of a mixed approach proved to be very efficient in large strain solid dynamics, circumventing the above-mentioned drawbacks for the traditional displacement based techniques. Early attempts at applying CFD-like numerical techniques in the context of displacement based computational solid dynamics are reported in references [2,43–48]. Eulerian Finite Volume Godunov methods, typically used for modelling compressible gas dynamics, were employed to model plastic flows in solid dynamics [49–51]. Furthermore, this methodology was also adapted to a Lagrangian framework by several authors [52,53], but restricted to two dimensions.

The use of a Total Lagrangian description of the motion has clear advantages in the context of solid dynamics. Firstly, all the calculations are carried out based on the undeformed mesh leading to a simple algorithm. Secondly, the Lagrangian setting follows the evolution of any material particle, which is of paramount importance in history dependent constitutive laws. Finally, the imposition of free surface boundary conditions is straightforward. On the contrary, the accuracy of the scheme can be adversely affected when undergoing very large deformations. This can be circumvented by employing adaptive remeshing techniques.

More recently, the $\boldsymbol{p}$–$\boldsymbol{F}$ formulation was improved in [5] for the case of nearly incompressible materials, by means of an additional conservation law for the Jacobian of the deformation J (widely known as volume map conservation law [42,54–57]), providing extra flexibility for the calculation of the volumetric stress. This innovative idea extended the range of use of the formulation to nearly and fully incompressible media. Moreover, further enhancement of the framework has recently been reported by the authors [58], when considering materials governed by a polyconvex constitutive law [59], enabling the symmetrisation of the resulting hyperbolic system of equations.

In this paper, the mixed $\boldsymbol{p}$–$\boldsymbol{F}$–J is discretised via an adapted upwind vertex centred Finite Volume Method (FVM) for linear tetrahedral meshes [60]. One clear advantage of using the upwind method is the ability to introduce physically-based numerical dissipation into the formulation derived from the Rankine–Hugoniot jump conditions. In addition, modern shock capturing techniques can be easily incorporated taking advantage of the conservative formulation. This can dramatically improve the performance of the algorithm in the vicinity of sharp spatial gradients. In this paper, a Total Variation Dimishing (TVD) space–time approach [3] is used, combining suitable slope limiters with a one-step two-stage explicit TVD Runge–Kutta time integrator.

Furthermore, the current paper extends the applicability of the formulation to include the consideration of thermo-mechanical processes. This requires the inclusion of the first law of thermodynamics (or known as conservation of the total energy E) and the satisfaction of the second law through the entropy production. The fully coupled mixed $\boldsymbol{p}$–$\boldsymbol{F}$–J–E system will then be supplemented with the simplest possible thermal-mechanical constitutive law for solids, namely Mie–Grüneisen equation of state [61]. For completeness and ease of understanding, the paper will present an eigenvalue analysis of the complete set of mixed system to ensure the satisfaction of the hyperbolicity, and thus material stability. A series of numerical examples will be examined to assess the robustness and capabilities of the mixed algorithm, yielding second order of convergence for velocities and stresses.

The outline of the present paper is as follows. Section 2 introduces a set of generalised governing equations for large strain non-isothermal fast dynamics, supplemented with appropriate mechanical constitutive models and equations of state. This section ends with the study of the eigenstructure of the problem. Section 3 describes the methodology of edge-based vertex centred FVM. Linear reconstruction, slope limiter and Riemann solver are also presented. Section 4 introduces the TVD Runge–Kutta time integrator used for temporal discretisation and some necessary numerical projections to preserve the angular momentum. Section 6 summarises the solution procedure of the proposed methodology. In Section 7, an extensive set of numerical examples is presented to assess the performance of the proposed method and to draw some comparisons against previous results published by the authors [3,5]. Finally, Section 8 summarises some concluding remarks and current directions of research.