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Modeling weakly-ionized plasmas in magnetic field: A new computationally-efficient approach

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ABSTRACT

Despite its success at simulating accurately both non-neutral and quasi-neutral weakly-ionized plasmas, the drift-diffusion model has been observed to be a particularly stiff set of equations. Recently, it was demonstrated that the stiffness of the system could be relieved by rewriting the equations such that the potential is obtained from Ohm's law rather than Gauss's law while adding some source terms to the ion transport equation to ensure that Gauss's law is satisfied in non-neutral regions. Although the latter was applicable to multicomponent and multidimensional plasmas, it could not be used for plasmas in which the magnetic field was significant. This paper hence proposes a new computationally-efficient set of electron and ion transport equations that can be used not only for a plasma with multiple types of positive and negative ions, but also for a plasma in magnetic field. Because the proposed set of equations is obtained from the same physical model as the conventional drift-diffusion equations without introducing new assumptions or simplifications, it results in the same exact solution when the grid is refined sufficiently while being more computationally efficient: not only is the proposed approach considerably less stiff and hence requires fewer iterations to reach convergence but it yields a converged solution that exhibits a significantly higher resolution. The combined faster convergence and higher resolution is shown to result in a hundredfold increase in computational efficiency for some typical steady and unsteady plasma problems including non-neutral cathode and anode sheaths as well as quasi-neutral regions.

1. Introduction

Generally referred to as magneto-plasmadynamics or magnetohydrodynamics (MHD), the process of applying a force on a fluid in motion using a magnetic field is the main mechanism behind several new aerospace technologies such as shockwave control in supersonic flows [1,2], power generation during re-entry using a MHD generator [3–5], heat shield in hypersonic flows [6,7], thrust generation using a Faraday accelerator [8,9], or efficiency improvement of pulse detonation engines through MHD energy bypass [10]. In such devices, the working fluid on which the magnetic field acts is air ionized either through high electric fields, through electron or microwave beams, or through potassium or cesium seeding.

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Independently of the ionization process, the ionization fraction of the air remains low (typically less than 0.1% or so) due to the energy needed to ionize the air being high relative to the flow enthalpy. For this reason, air plasmas in aerospace applications can be considered *weakly-ionized*.

When assuming quasi-neutrality throughout, the numerical simulation of weakly-ionized plasmas in magnetic field can be accomplished efficiently by obtaining the potential from the generalized Ohm's law (see Refs. [4,9] for instance). However, because the quasi-neutral assumption limits its use to plasmas in which the positive and negative charges closely approach each other, such a strategy cannot be applied in the vicinity of dielectric surfaces or within the cathode and anode sheaths where the positive charge density differs substantially from the negative charge density. Because the accurate modelling of the non-neutral regions near the surfaces is often critical due to the large voltage (and hence power) drop within cathode sheaths, the numerical simulation of many weakly-ionized plasmas cannot be accomplished through the generalized Ohm's law without inducing excessive error in the solution. Rather, it is deemed necessary for many problems to obtain the potential from Gauss's law and to solve additional transport equations to account for the motion of the ions and the electrons with respect to the neutrals. Commonly referred to as the "drift-diffusion model", such a strategy was first demonstrated viable in solving weakly-ionized gases under the influence of an externally-applied magnetic field in Ref. [11], and was used subsequently to obtain multiple solutions of gas discharges in which the electrons were magnetized (see for instance Refs. [12–14]).

Despite its success at predicting accurately both non-neutral and quasi-neutral plasmas in the presence of magnetic field, the drift-diffusion model has been observed to be an exceptionally stiff set of equations. That is, the system of equations is such that it forces a numerical method to use an integration steplength which is excessively small in relation to the smoothness of the exact solution, hence resulting in a disproportionate number of iterations to reach convergence. The stiffness is further exacerbated should the plasma contain quasi-neutral regions of substantial size, in which case the number of iterations needed to obtain a solution is in the order of millions. In Ref. [15], it was argued that the stiffness of the drift-diffusion model originates from the potential equation based on Gauss's law being particularly sensitive to small errors in the charged species densities when the plasma becomes quasi-neutral. It was then demonstrated that the stiffness of the system could be relieved by rewriting the equations such that the potential is obtained from Ohm's law rather than Gauss's law while adding some source terms to the ion transport equation to ensure that Gauss's law is satisfied in non-neutral regions (see Ref. [15] and also Ref. [16]).

The recast of the drift-diffusion set of equations first proposed in Ref. [15] was extended to multicomponent and multidimensional plasmas in Ref. [17], where several test cases involving quasi-neutral plasmas between dielectrics and non-neutral discharges between electrodes showed a remarkable improvement in computational efficiency compared to the conventional approach: Not only did the recast set of equations permit the use of considerably higher integration steplengths resulting in a thirtyfold or more reduction in the number of iterations to reach convergence, but it also resulted in a higher resolution of the converged solution whenever the plasma included quasi-neutral regions of substantial size. The combined gains in resolution and convergence rates resulted in the recast system of equations being typically 100 times more computationally efficient than the conventional drift-diffusion equations while not sacrificing on the generality of the physical model.

Despite being generally applicable to weakly-ionized plasmas in multiple dimensions including plasmas with various types of ions (including negative ions), the recast set of transport equations presented in Ref. [17] is not applicable to a plasma in which either the ions or the electrons are magnetized and can hence not be used to solve problems in which the external magnetic field is significant. The goal of this paper is hence to craft a new computationally-efficient set of electron and ion transport equations that can be used not only for a multicomponent and multidimensional plasma, but also for a plasma in magnetic field. As will be shown subsequently, this will require the potential equation to be based on the *generalized* Ohm's law rather than the standard form of Ohm's law and to require a change in the definition of the ambipolar electric field when recasting the transport equations for the negatively-charged species. As in prior work, it is ensured that the recast set of equations is obtained from the same physical model as the conventional drift-diffusion equations and, as such, yields the same exact solution either within quasi-neutral regions or within non-neutral regions including cathode, anode, and dielectric sheaths.

2. Physical model

Let us now outline the physical model from which the recast computationally-efficient set of transport equations will be subsequently derived. Commonly referred to as the "fluid model" or "drift-diffusion model", the physical model under consideration treats the neutrals and each charged species as independent fluids with their own velocities interacting with the other fluids through collision forces. In the presence of a magnetic field, the drift-diffusion model yields the following mass conservation equation for each charged species (either electrons, positive ions, or negative ions):

$$\frac{\partial N_k}{\partial t}+\sum_{i=1}^{3}\frac{\partial}{\partial x_i}N_k\boldsymbol{V}_i^k=W_k \tag{1}$$

where k is an index associated with the species to be solved and where the species velocity $\boldsymbol{V}^k$ is obtained from:

$$\boldsymbol{V}^k=\boldsymbol{V}^{\rm n}+s_k\mu_k\left(\boldsymbol{E}+\boldsymbol{V}^k\times\boldsymbol{B}\right)-\frac{\mu_k}{|C_k|N_k}\nabla P_k \tag{2}$$