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Perfectly matched layer absorbing boundary condition for nonlinear two-fluid plasma equations

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ABSTRACT

Numerical instability occurs when coupled Maxwell equations and nonlinear two-fluid plasma equations are solved using finite difference method through parallel algorithm. Thus, a perfectly matched layer (PML) boundary condition is set to avoid the instability caused by velocity and density gradients between vacuum and plasma. A splitting method is used to first decompose governing equations to time-dependent nonlinear and linear equations. Then, a proper complex variable is used for the spatial derivative terms of the time-dependent nonlinear equation. Finally, with several auxiliary function equations, the governing equations of the absorbing boundary condition are derived by rewriting the frequency domain PML in the original physical space and time coordinates. Numerical examples in one- and two-dimensional domains show that the PML boundary condition is valid and effective. PML stability depends on the absorbing coefficient and thickness of absorbing layers.

1. Introduction

In nuclear fusion, solving coupled Maxwell equations and nonlinear two-fluid plasma equations in full space–time domain using a finite difference method is challenging. On the one hand, a set of real boundary conditions at the plasma edge is difficult to provide. For instance, the velocities and disturbed densities of bounded and inhomogeneous plasma cannot be accurately established based on the separatrix of plasma and vacuum. On the other hand, the existence of the nonlinear terms of fluid equations at the boundary results in numerical oscillation because of the large velocity and density gradients of plasma caused by the applied electromagnetic wave. Therefore, the calculation cannot be performed. If the velocities and disturbed densities of the electrostatic wave at the plasma edges are sufficiently small, numerical experiments could prove that the divergence can be suppressed effectively, and the numerical reflection of the electrostatic wave could be avoided at the numerical boundary. Hu et al. showed that traditional techniques, such as damping and filtering, cannot be applied [1,2]. Especially for the simulations of parametric decay instabilities (PDIs), long times and space iterations are necessary to obtain evolutions, which amplifies the limitation of numerical instability. Thus, another type of artificial boundary layer is required to smoothly reduce the physical quantities of the electrostatic component to an acceptable degree. Perfectly matched layer (PML) could provide an excellent approach to remedy this problem.

PML was first proposed by Berenger in 1994 to absorb outgoing wave in open computational boundaries for electromagnetic computations [3,4]. This method is significant because the absorbing zone theoretically does not allow reflection of

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waves at any angle and frequency. This method was optimized and derived in polar coordinates by Collino and Monk (1998) [5], whereas Gedney (1996) showed that PML technique is actually equivalent to a complex change in the variables in the frequency domain [6]. Non-split technique, another type of PML formulation, was developed by utilizing a complex change of variables in space [7–13]. However, certain numerical instability was observed through PML [14,15]. Split and non-split formulations were all proven to be weakly posed [7,10,13,15]. Further studies have indicated that all absorbed physical waves should have consistent phase and group velocities in a PML [7,11,13]. To satisfy this prerequisite, proper space–time transformation should be considered to guarantee that PML would function well [7,11].

Given its superior performance and wide application, PML had been widely used for boundary problems of numerical simulations for numerous types of physical equations in the past two decades. PML was first applied by Hu to treat boundaries of the linear Euler equation with uniform and non-uniform mean flow [11,14]. Thereafter, the performance of this method has been extensively improved to solve linear Euler equation systems [13,16–20]. Collino (1997) used this technique to treat paraxial equations [21]. Singer and Turkel (2004) developed a Helmholtz-PML formation to solve the Helmholtz equation of an infinite two-dimensional (2D) strip [22]. Gondarenko et al. (2004) solved time-dependent Maxwell's equations using PML to study the Kelvin–Helmholtz instability of anisotropic inhomogeneous ionosphere plasmas [23]. Zheng (2007) and Dohnal (2009) utilized PML to study nonlinear Schrodinger wave equations [24,25]. Dohnal and Hagstrom (2007) computed for the PML boundary of nonlinear-coupled mode equations [26]. Hu et al. utilized PML to compute for the nonlinear Euler and Navier–Stokes equations [8,9,12,27,28]. Lantos and Nataf (2010) used PML to solve heat and advection–diffusion equations [29]. Thus, PML can be efficiently applied to various types of partial differential mathematical equations.

However, the application of PML boundary conditions on nonlinear two-fluid plasma equations has not yet been reported. For this case, PML application seems more complicated. For the electromagnetic wave just outside the plasma, plasma oscillation velocity produced by the incident electromagnetic always exists at the entrance. To use PML to reduce the velocity gradient related with the numerical oscillation between plasma and vacuum, the plasma velocity should first be divided into two parts, namely, the linear electromagnetic components derived from the linear moment equations of plasma, and the part containing all nonlinear coupled terms and electrostatic components that causes the numerical divergence. Then, only the part with the nonlinear equation will be absorbed by PML. In this study, PML is mainly realized through a complex variable change. Last, PML formulations with several introduced auxiliary function equations are derived through a transformation from the frequency domain to time domain by numerically solving the aforementioned equations. PML is implemented to reduce numerical oscillation.

The remainder of this paper is organized as follows. In Section 2, a nonlinear fluid plasma model and numerical methods of coupled equations are introduced. In Section 3, the formulation of the PML absorbing boundary condition for nonlinear two-fluid plasma equations is presented. In Section 4, numerical tests were performed for the PML boundary conditions with the laser nuclear fusion parameters of plasma and field, as well as the benchmarking of the code through the analytical linear theory of several PDI types. Conclusions are drawn in Section 5.

2. Physical equations and numerical algorithm

2.1. Physical equations

Propagation equations of an electromagnetic wave in plasma produced by an antenna are given by

$$\nabla \times \boldsymbol{E} = -\mu_0 \frac{\partial \boldsymbol{H}}{\partial t} \tag{1}$$

$$\nabla \times \boldsymbol{H} = \varepsilon_0 \frac{\partial \boldsymbol{E}}{\partial t} + \boldsymbol{J} \tag{2}$$

where $\boldsymbol{J}$ is the current density in plasma

$$\boldsymbol{J} = \sum_{\alpha} q_\alpha n_\alpha \boldsymbol{u}_\alpha \tag{3}$$

The subscript α is the particle species (such as electrons or ions). q_α, n_α, and $\boldsymbol{u}_\alpha$ are the particle charge, plasma density, and velocity, respectively. $n_\alpha = n_{\alpha 0} + \delta n_\alpha$, where $n_{\alpha 0}$ is the equilibrium density, and δn_α is the disturbance density of the plasma.

Continuity and momentum equations are expressed by

$$\frac{\partial n_\alpha}{\partial t} + \nabla \cdot (n_\alpha \boldsymbol{u}_\alpha) = 0 \tag{4}$$

$$\frac{\partial \boldsymbol{u}_\alpha}{\partial t} + \boldsymbol{u}_\alpha \cdot \nabla \boldsymbol{u}_\alpha = \frac{q_\alpha}{m_\alpha}\left(\boldsymbol{E} - \nabla \Psi + \frac{\boldsymbol{u}_\alpha \times \boldsymbol{B}}{c}\right) - \frac{\gamma_\alpha T_\alpha}{m_\alpha}\frac{1}{n_\alpha}\nabla \delta n_\alpha \tag{5}$$

where γ_α is the ratio of specific heats, $\boldsymbol{B}$ is the disturbing magnetic field of electromagnetic wave, and Ψ is the wave potential, which can be described by Poisson's equation

$$\nabla^2 \Psi = -e(\delta n_i - \delta n_e)/\varepsilon_0 \tag{6}$$