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# Asymptotic diffusion limit of cell temperature discretisation schemes for thermal radiation transport

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ABSTRACT

This paper attempts to unify the asymptotic diffusion limit analysis of thermal radiation transport schemes, for a linear-discontinuous representation of the material temperature reconstructed from cell centred temperature unknowns, in a process known as 'source tilting'. The asymptotic limits of both Monte Carlo (continuous in space) and deterministic approaches (based on linear-discontinuous finite elements) for solving the transport equation are investigated in slab geometry. The resulting discrete diffusion equations are found to have nonphysical terms that are proportional to any cell-edge discontinuity in the temperature representation. Based on this analysis it is possible to design accurate schemes for representing the material temperature, for coupling thermal radiation transport codes to a cell centred representation of internal energy favoured by ALE (arbitrary Lagrange–Eulerian) hydrodynamics schemes.



## 1. Introduction

High-energy density physics (HEDP) calculations to simulate experiments, such as inertial confinement fusion or high Mach number shock interactions, require the combination of two disparate fields of computational physics: hydrodynamics and thermal radiation transport. Computational hydrodynamics and radiation transport each have unique approaches to numerically solving the constituent partial differential equations. For example, radiation transport methods, in general, need discretisations designed to preserve the asymptotic diffusion limit of transport, often requiring several unknowns per spatial cell. On the other hand, hydrodynamics methods often use a single degree of freedom per spatial cell, which can lead to unexpected difficulties when coupling the two methods. This paper focuses on one particular aspect of this coupling: the coupling of hydrodynamics codes that use a cell centred treatment of the internal (material) energy to Monte Carlo and deterministic solutions of time-dependent thermal radiation transport problems. In such an arrangement, the corresponding first-order accurate representation of the material temperature is insufficient to facilitate accurate computation of the thermal energy transport processes for algorithms which properly account for the directionality of the radiation field when the computational mesh does not resolve the radiation mean-free path.

Discrete diffusion schemes also need to make use of a higher order representation of the material temperature in order to avoid problems with excess numerical diffusion. However, this can be overcome by using a mixed element formulation which decomposes the second order diffusion equation into a set of coupled first order equations. A number of schemes

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exist which produce accurate solutions to this set of equations based on edge-based representations of the normal component of the flux vector [1,2].

Deterministic transport algorithms based on discontinuous finite element formulations perform best if they use a nodal representation of the material temperature field, rather than the piecewise constant representation inherited from the hydrodynamics algorithm, which produces first order convergence with respect to the mesh spacing. In addition to recovering a second order convergence rate for sufficiently fine meshes, this paper demonstrates that a nodal treatment is essential in order to ensure accurate results in the asymptotic diffusion limit for both deterministic and Monte Carlo transport algorithms.

The asymptotic diffusion limit refers to meshes which are sufficiently fine that they resolve the transport processes on the coarser length-scale associated with the diffusion of radiation energy through opaque media, but where it is impractical to resolve down to a photon mean-free path. Schemes which do not perform well in this limit are generally unsuitable for modelling problems such as laser hohlraums, where the optical depth of the walls and the capsule are so large that it is impractical to use meshes with more than a few tens of cells through their respective thicknesses, especially for 3D simulations.

While our focus is on deterministic schemes based on the use of discontinuous finite elements for modelling the spatial variations, we acknowledge that it is possible to develop accurate schemes for solving the transport equation based on enforcing particle balance in local sub-volumes. The family of corner balance transport schemes splits a mesh of arbitrary polygonal elements into a set of sub-cells (corners) and couples the transport equation in each sub-volume to a set of independent piecewise constant material temperature values [3]. This approach is similar in spirit to the use of node centred temperatures, as there is still the requirement to model the variation of the material temperature within each sub-element in order to produce accurate results in the diffusion limit. Another way for deterministic transport algorithms to implement a nodal representation of the temperature in practice is to have the radiation solver "own" a slope of the temperature in each cell that is not communicated to the hydrodynamics scheme [4]. This arrangement can lead to discontinuities in the temperature representation.

We analyse the asymptotic diffusion limit of both deterministic and Monte Carlo transport schemes in slab geometry, illustrating the close correspondence between the results. This leads us towards the formulation of discrete diffusion equations for the mesh cells which are approximations to the energy balance equation satisfied by Monte Carlo transport schemes and deterministic transport discretisations based on the use of linear-discontinuous finite elements.

These discrete equations allow us to synthesize the results of the transport solutions on coarse meshes, as well as highlighting the most important aspects of the algorithm used to construct a higher order representation of material temperature suitable for use in the transport simulations. This theoretical underpinning is essential in order to guide the search for improved tilt schemes towards the most profitable avenues for future research.

Initially we consider the behaviour of a purely absorbing medium. This analysis is then extended to consider scattering problems, where the scattering is assumed to be a consequence of the semi-implicit temporal discretisation techniques which are commonly employed to overcome the time-step constraints due to the stiff coupling associated with the energy exchange term. These techniques are observed to improve the spatial convergence behaviour and this is confirmed by the asymptotic analysis. Unfortunately they also degrade the temporal accuracy of the results, leading to an interplay between the spatial and temporal resolution which can produce misleading results in convergence studies.

## 2. Problem description

We consider the solution of the coupled thermal radiation transport equation [5],

$$\frac{1}{c}\frac{\partial \Psi}{\partial t} + \mu \frac{\partial \Psi}{\partial x} + \sigma \Psi = \sigma \Phi \tag{1a}$$

and material energy equation,

$$(\rho C_V)\frac{\partial T}{\partial t} = 2\pi \int_{-1}^{1} \sigma (\Psi - \Phi)\, d\mu \tag{1b}$$

for a purely absorbing slab in a grey medium. In these equations, $x$ is the spatial variable, $t$ time variable, $\mu$ is the cosine of the angle between a photon's direction-of-flight and the $x$-axis, $\Psi(\mu, x, t)$ is the intensity of thermal radiation, $\sigma(x, t)$ is the absorption opacity, $c$ is the speed of light. The material temperature is denoted by $T(x, t)$, and $\rho C_V(x, t)$ is the heat capacity (the material density times the specific heat). The quantity $\Phi(x, t) \equiv \Phi(T(x, t))$ is the thermal emissivity, which is a Planckian blackbody emission source integrated over all frequency:

$$\Phi(T) = \frac{acT^4}{4\pi} \tag{2}$$

where $a = 4\sigma_{sb}/c$ ($\sigma_{sb}$ is the Stefan–Boltzmann constant).

In the asymptotic diffusion limit this reduces to a single equilibrium diffusion equation [6],