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A high-precision calculation method for interface normal and curvature on an unstructured grid

Kei Ito [a,*], Tomoaki Kunugi [b], Shuji Ohno [a], Hideki Kamide [a], Hiroyuki Ohshima [a]

[a] *Advanced Nuclear System Research and Development Directorate, Japan Atomic Energy Agency, 4002 Narita, O-arai, Ibaraki 311-1393, Japan*
[b] *Department of Nuclear Engineering, Kyoto University, Yoshida, Sakyo, Kyoto 606-8501, Japan*

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ABSTRACT

In the volume-of-fluid algorithm, the calculations of the interface normal and curvature are crucially important for accurately simulating interfacial flows. However, few methods have been proposed for the high-precision interface calculation on an unstructured grid. In this paper, the authors develop a height function method that works appropriately on an unstructured grid. In the process, the definition of the height function is discussed, and the high-precision calculation method of the interface normal is developed to meet the necessary condition for a second-order method. This new method has highly reduced computational cost compared with a conventional high-precision method because the interface normal calculation is completed by solving relatively simple algebraic equations. The curvature calculation method is also discussed and the approximated quadric curve of an interface is employed to calculate the curvature. Following a basic verification, the developed height function method is shown to successfully provide superior calculation accuracy and highly reduced computational cost compared with conventional calculation methods in terms of the interface normal and curvature. In addition, the height function method succeeds in calculating accurately the slotted-disk revolution problem and the oscillating drop on unstructured grids. Therefore, the developed height function method is confirmed to be an efficient technique for the high-precision numerical simulation of interfacial flows on an unstructured grid.

1. Introduction

The volume-of-fluid algorithm [1,2] is a well-known interface-tracking method, in which the following transport equation of volume fraction is solved to simulate interfacial dynamic phenomena:

$$\frac{\partial f}{\partial t} + \boldsymbol{v} \cdot \nabla f = 0, \qquad (1)$$

where f is the volume fraction ranging from zero to unity and $\boldsymbol{v}$ is the velocity vector. The volume fraction indicates the ratio of a liquid phase in each computational cell; therefore, an interface is located in a cell when f is between zero and unity. The volume-of-fluid algorithm is widely used in scientific and industrial simulations of interfacial flows.

* Corresponding author. Tel.: +81 29 267 4141; fax: +81 29 266 3675.
E-mail address: ito.kei@jaea.go.jp (K. Ito).

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The authors have developed a high-precision simulation code with the volume-of-fluid algorithm to simulate interfacial dynamic phenomena, e.g., gas entrainment [3–6] in nuclear reactors. In this simulation code, the concept of the piecewise linear interface calculation (PLIC) [7,8] is employed, in which the interface normal is taken into account to calculate Eq. (1) accurately. Thus, the precise evaluation of the interface normal is most important in PLIC-like simulations.

For the interface normal calculation in a PLIC-like algorithm, various methods have been proposed. The conventional Parker–Youngs method [9] uses a relatively simple difference formula of volume fraction to calculate the gradient of the volume fraction, which is equivalent to the interface normal. An application of this method to unstructured grids is not difficult and several simulation codes have been developed, e.g. [10]. However, the Parker–Youngs method is at most a first-order method, because the interface normal of a rectilinear interface is not always calculated precisely. This condition for the calculation of rectilinear interfaces is well known as a necessary condition for a second-order method [11]. The least squares fit [12,13], which approximates the interfacial shape by a linear line or quadratic curve, is a much more accurate method compared to the Parker–Youngs method. Furthermore, the least squares fit method has been extended to unstructured grids [14]. However, even with the least squares fit methods, the precise interface normal of a rectilinear interface is not always obtained.

To enhance the calculation accuracy of the interface normal, several second-order methods have been proposed. The least squares volume-of-fluid interface reconstruction algorithm (LVIRA) [11] is one such second-order method, in which the location of a linear interface is determined in an interfacial cell to be consistent with the volume fraction distribution around the interfacial cell. In this procedure, the discrepancy between the calculated linear interface and the volume fraction distribution is minimized in the sense of the least squares fit. LVIRA requires an iterative geometric calculation (for the interface reconstruction), and, therefore, the computational cost is high. The efficient least squares volume-of-fluid interface reconstruction algorithm (ELVIRA) [11] has been therefore proposed to reduce the computational cost of LVIRA. The calculation concept of ELVIRA is the same as LVIRA, i.e., the interface is determined to be consistent with the volume fraction distribution. In ELVIRA, however, the interface normal is chosen from several candidates given by conventional methods; therefore, the interface reconstruction is not conducted iteratively.

A similar method to LVIRA, called geometric least squares (GLS) [15] has been proposed for unstructured grids. GLS gives an accurate calculation result for the interface normal, but the computational cost can be very high. LVIRA, ELVIRA, and GLS are guaranteed to reproduce any rectilinear interfaces. Therefore, these methods are called second-order methods. Mosso et al. [16] proposed a method with a global (the whole computational domain) iterative calculation. In this method, the discontinuity of the interfaces in adjacent cells is minimized on the boundary of the adjacent cells. It is obvious that this concept can reproduce any rectilinear interface, even on an unstructured grid, but the global iterative calculation has a significantly high computational cost.

Another well-known second-order scheme is the height function method [17], which has been originally developed for the curvature calculation. In the height function method, an interface normal is calculated based on the difference formula of the height function. On a structured grid, the height function is defined by integrating the volume fraction in a vertical or horizontal direction. On an unstructured grid, however, such integration is very difficult because the area for integration could not be determined appropriately due to irregular cell arrangement, and the integral calculation can be extremely complicated. Therefore, to the best of the authors' knowledge, the height function method has not been proposed for an unstructured grid.

The curvature calculation is also important in the volume-of-fluid algorithm to evaluate the surface tension. In general, the curvature is calculated based on the volume fraction distribution, i.e., the curvature can be given as the second derivative of the volume fraction distribution. However, an accurate calculation of the second derivative is not easy because the volume fraction is a Heaviside function, not a differentiable function. Therefore, in the traditional continuum surface force (CSF) method [18], a convolution of the volume fraction with a kernel function is performed to make a smooth function of the volume fraction distribution, and then the curvature is calculated as the second derivative of the smooth function. The CSF method is widely used on both structured and unstructured grids.

To enhance the accuracy of the curvature calculation, the reconstructed distance function (RDF) method [19] has been proposed, in which the distance function from an interface is defined in each cell to calculate the curvature as the second derivative of the distance function distribution (smooth function). The authors have been extended the RDF method to an unstructured grid [20].

A more accurate curvature calculation can be achieved using the height function method [21]. In this method, the height function is defined as described above, and the curvature is calculated as the second derivative of the height function distribution. It is known that the height function method gives second-order accuracy in some simple problems. However, the height function method for the curvature calculation has not been proposed for an unstructured grid.

In this paper, the authors develop a height function method for the interface normal calculation that works appropriately on an unstructured grid which consists of triangular cells and/or quadrilateral cells. First, a way to define the height function is discussed, and then the geometric relationship equation of the height function is considered to calculate the interface normal accurately. This new method is guaranteed to reproduce any rectilinear interfaces, and therefore a second-order method. In addition, the height function is also employed for the curvature calculation, in which an approximated quadratic curve of the interface is determined by minimizing the discrepancy between the quadratic curve and the interface in the sense of the least squares fit. The curvature is given as the formula of the coefficients of the quadratic curve. To verify the developed height function method (called the unstructured height function method in this paper), several simple problems