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Stable pseudoanalytical computation of electromagnetic fields from arbitrarily-oriented dipoles in cylindrically stratified media

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ABSTRACT

Computation of electromagnetic fields due to point sources (Hertzian dipoles) in cylindrically stratified media is a classical problem for which analytical expressions of the associated tensor Green's function have been long known. However, under finite-precision arithmetic, direct numerical computations based on the application of such analytical (canonical) expressions invariably lead to underflow and overflow problems related to the poor scaling of the eigenfunctions (cylindrical Bessel and Hankel functions) for extreme arguments and/or high-order, as well as convergence problems related to the numerical integration over the spectral wavenumber and to the truncation of the infinite series over the azimuth mode number. These problems are exacerbated when a disparate range of values is to be considered for the layers' thicknesses and material properties (resistivities, permittivities, and permeabilities), the transverse and longitudinal distances between source and observation points, as well as the source frequency. To overcome these challenges in a systematic fashion, we introduce herein different sets of range-conditioned, modified cylindrical functions (in lieu of standard cylindrical eigenfunctions), each associated with nonoverlapped subdomains of (numerical) evaluation to allow for stable computations under any range of physical parameters. In addition, adaptively-chosen integration contours are employed in the complex spectral wavenumber plane to ensure convergent numerical integration in all cases. We illustrate the application of the algorithm to problems of geophysical interest involving layer resistivities ranging from 1000 Ω m to 10^{-8} Ω m, frequencies of operation ranging from 10 MHz down to the low magnetotelluric range of 0.01 Hz, and for various combinations of layer thicknesses.

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1. Introduction

Computation of electromagnetic fields due to arbitrarily-oriented elementary (Hertzian) dipoles in cylindrically stratified media is of interest in a wide range of scenarios, including geophysical exploration, fiber optics, and radar cross-section analysis. Assuming the z-axis to be the symmetry axis, the analytical formulation of this problem is predicated on the knowledge of the cylindrical eigenfunctions (Bessel and Hankel functions) in the domain transverse to z and their modal amplitudes.

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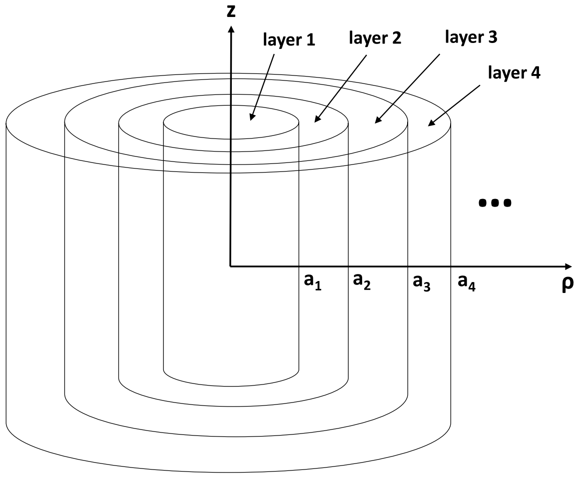

Fig. 1. Basic geometry of a cylindrically stratified medium.

The derivation of reflection and transmission coefficients at each cylindrical boundary is then ascertained through the use of the proper boundary conditions. Since the eigenfunctions comprise a continuum spectrum in an unbounded domain, a Fourier-type integral along the spectral wavenumber k_z is subsequently necessary to determine the fields (tensor Green's function) [1–7]. Unfortunately, numerical computations based on direct use of such (canonical) analytical expressions often lead to underflow and/or overflow problems under finite-precision arithmetic. These problems are related to the poor scaling of cylindrical eigenfunctions for extreme arguments and/or high-order, as well as convergence problems related to the numerical evaluation of the spectral integral on k_z and truncation of the infinite series over the azimuth mode number n. Underflow and overflow problems become especially acute when a disparate range of values needs to be considered for the physical parameters, viz., the layers' constitutive properties (resistivities, permittivities, and permeabilities) and thicknesses, the transverse and longitudinal distance between source and observation points, as well as the source frequency [8]. In order to stabilize the numerical computation in the case of plane-wave scattering by highly absorbing layers, Swathi and Tong [9] developed an algorithm based upon scaled cylindrical functions. A stabilization procedure to deal with a very large number of cylindrical layers and disparate radii was proposed in [10]. Similar issues appear when computing Mie scattering from multilayered spheres [11,12], where continued fractions [13] or logarithmic derivatives [14], for example, can be used to circumvent the recurrence instability of Bessel functions of very large order. When the overall computation cost is not an issue, a more extreme strategy to circumvent this problem is to use arbitrary-precision arithmetic [15].

In this work, we extend those efforts by considering the stable computation of electromagnetic fields due to points sources in cylindrically stratified media. A salient feature of our work is that we do not limit ourselves to a particular regime of interest (that is very small or very large radii, or highly absorbing layers) but instead develop a systematic algorithm to enable stable computations in any scenario. Note that, as opposed to Mie scattering or plane-wave scattering from cylinders considered in the above, the required spectral integration over k_z produces, *per se*, a large variation on the integrand function arguments. The numerical convergence of such spectral integral, which depends among other factors on the separation between source and observation points, need to be considered in tandem with the numerical stabilization procedure. Our methodology is based on the use of various sets of range-conditioned, modified cylindrical functions (in lieu of standard cylindrical eigenfunctions), each evaluated in nonoverlapped subdomains to yield stable computations under double-precision floating-point format for any range of physical parameters. This is combined with different numerically-robust integration contours that are adaptively chosen in the complex k_z plane to yield fast convergence. We illustrate the algorithm in problems of geophysical interest involving layer resistivities ranging from 1000 $\Omega\,$m to about 10^{-8} $\Omega\,$m, frequencies ranging from 10 MHz to as low as 0.01 Hz, and for various layer thicknesses.

2. Range-conditioned formulation

Fig. 1 shows the geometry of a cylindrically stratified medium. Hereinafter we shall refer to the layer where the point source is present as layer j and the region where the fields are computed as layer i. As indicated in Fig. 1, each successive layer radius is denoted by a_i.

We use nonprimed coordinates (ρ, ϕ, z) for the observation point and primed coordinates (ρ', ϕ', z') for the source location, $k_{i\rho} = (k_i^2 - k_z^2)^{1/2}$ denotes the transverse wavenumber in layer i and k_z is the longitudinal wavenumber, where $k_i^2 = \omega^2 \mu_i \epsilon_i$. As usual, ω represents the angular frequency, $\mu_i = \mu_{r,i} + \mathrm{i}\sigma_{m,i}/\omega$ the complex permeability, and $\epsilon_i = \epsilon_{r,i} + \mathrm{i}\sigma_i/\omega$ the complex permittivity, with $\epsilon_{r,i}$, $\mu_{r,i}$ denoting the real-valued permittivity and permeability, resp., and σ_i, $\sigma_{m,i}$ denoting

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