Journal of Computational Physics 273 (2014) 472–487

Contents lists available at ScienceDirect

Journal of Computational Physics

www.elsevier.com/locate/jcp

Ground detection by a single electromagnetic far-field measurement

Jingzhi Li [a,*], Hongyu Liu [b], Yuanchang Sun [c], Qi Wang [d]

[a] Faculty of Science, South University of Science and Technology of China, 518055 Shenzhen, China
[b] Department of Mathematics and Statistics, University of North Carolina, Charlotte, NC 28223, USA
[c] Department of Mathematics and Statistics, Florida International University, Miami, Florida 33199, USA
[d] Department of Computing Sciences, School of Mathematics and Statistics, Xi'an Jiaotong University, Xi'an 710049, China

ARTICLE INFO

Article history:
Received 8 January 2014
Received in revised form 15 May 2014
Accepted 21 May 2014
Available online 27 May 2014

Keywords:
Inverse electromagnetic scattering
Multiscale ground objects
Locating
Indicator functions
A single measurement

ABSTRACT

We consider detecting objects on a flat ground by using the electromagnetic (EM) measurement made from a height. Our study is conducted in a very general and practical setting. The number of the target scatterers is not required to be known in advance, and each scatterer could be either an inhomogeneous medium or an impenetrable perfectly conducting (PEC) obstacle. Moreover, there might be multiscale components of small-size and extended-size (compared to the detecting wavelength) presented simultaneously. Some a priori information is required on scatterers of extended-size. The inverse problem is nonlinear and ill-conditioned. We propose a "direct" locating method by using a single EM far-field measurement. The results extend those obtained in [17,18] for locating multiscale EM scatterers located in a homogeneous space.

1. Introduction

This paper concerns locating objects on a ground by using the electromagnetic (EM) scattering measurement made from a height. In Fig. 1, we give a schematic illustration of our study, where one wants to detect the multiple objects on the ground $\mathscr{G}$. To that end, one sends certain detecting wave fields and then measures the scattered wave fields from a height, from which to infer knowledge about the target objects. A practical scenario for our study is the scoutplane detection in the battlefield.

In what follows, we present the mathematical formulation for the current study. The detecting waves are chosen to be the time-harmonic electromagnetic plane waves of the following form

$$E^i(x) = pe^{\mathrm{i}\omega x\cdot d}, \qquad H^i(x) = \frac{1}{\mathrm{i}\omega}\nabla\wedge E^i(x), \quad x\in\mathbb{R}^3 \tag{1.1}$$

where $\mathrm{i}=\sqrt{-1}$, $\omega\in\mathbb{R}_+$ denotes the frequency, $d\in\mathbb{S}^2 := \{x\in\mathbb{R}^3; |x|=1\}$ denotes the impinging direction, and $p\in\mathbb{R}^3$ denotes the polarization with $p\cdot d=0$. E^i and H^i are entire solutions to the Maxwell equations in the free space

$$\nabla\wedge E^i - \mathrm{i}\omega H^i = 0, \qquad \nabla\wedge H^i + \mathrm{i}\omega E^i = 0.$$

* Corresponding author.
E-mail addresses: li.jz@sustc.edu.cn (J. Li), hongyu.liuip@gmail.com (H. Liu), yuasun@fiu.edu (Y. Sun), qi.wang.xjtumath@gmail.com (Q. Wang).

http://dx.doi.org/10.1016/j.jcp.2014.05.027

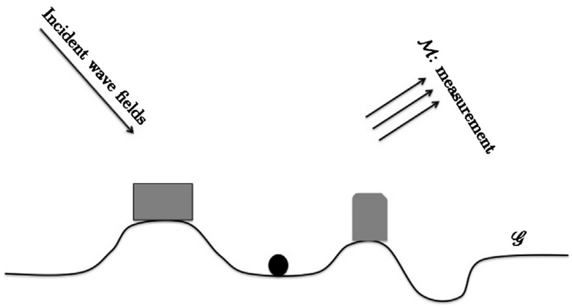

Fig. 1. Schematic illustration of locating multiscale ground objects.

The ground $\mathscr{G}$ is assumed to be perfectly electric conducting (PEC). The EM waves cannot penetrate inside the ground and propagate only in the space above the ground. If there is no object presented on the ground, one would have a reflected wave field $E^i_{\mathscr{G}}$ such that the total wave field $E = E^i - E^i_{\mathscr{G}}$ satisfies the following PEC boundary condition

$$\nu \wedge E = \nu \wedge \left(E^i - E^i_{\mathscr{G}}\right) = 0 \quad \text{on } \mathscr{G}, \tag{1.2}$$

where ν is the unit upward normal vector to $\mathscr{G}$. If $\mathscr{G}$ is flat, the reflected wave field $E^i_{\mathscr{G}}$ is well-understood through the work [22,23], and if $\mathscr{G}$ is non-flat/rough, the reflection would be much more complex. Throughout the present work, we assume that $\mathscr{G}$ is flat and shall leave the rough case for a future study. Furthermore, without loss of generality, we assume that $\mathscr{G} := \{x := (x_1, x_2, x_3) \in \mathbb{R}^3; x' := (x_1, x_2) \in \mathbb{R}^2,\ x_3 = 0\}$. Denote $\mathbb{R}^3_\pm := \{x := (x_1, x_2, x_3) \in \mathbb{R}^3; x' := (x_1, x_2) \in \mathbb{R}^2,\ x_3 \gtrless 0\}$ and $\mathbb{S}^2_\pm := \mathbb{R}^3_\pm \cap \mathbb{S}^2$. Moreover, we let Π denote the usual reflection with respect to $\mathscr{G}$, i.e., $\Pi v = (v_1, v_2, -v_3)$ for a generic 3-vector $v = (v_1, v_2, v_3)$. Then, we have that (cf. [22,23])

$$E^i_{\mathscr{G}} = \Pi \circ E^i \circ \Pi. \tag{1.3}$$

Next, we consider that there are EM objects presented on the ground. Let $\psi(x')$, $x' \in \mathbb{R}^2$, be a non-negative Lipschitz continuous function such that $\psi(x') = 0$ for $|x'| > R$, where R is a large enough positive constant. Let $\Sigma' := \{x' \in \mathbb{R}^2; \psi(x') > 0\}' := \bigcup_{j=1}^{l} \Sigma'_j$, where Σ'_j, $j = 1, 2, \ldots, l$ denote the simply connected components of Σ'. Define

$$\Sigma_j^+ := \left\{\left(x', x_3\right) \in \mathbb{R}^3_+; x' \in \Sigma'_j,\ 0 < x_3 < \psi\left(x'\right)\right\}, \quad 1 \le j \le l; \qquad \Sigma^+ := \bigcup_{j=1}^{l} \Sigma_j^+. \tag{1.4}$$

Each Σ_j^+, $1 \le j \le l$, represents an EM object on the ground, and will be referred to as a *scatterer* in the sequel. Let ε_j, μ_j and Σ_j be the EM parameters for the object supported in Σ_j, respectively, representing the electric permittivity, magnetic permeability and electric conductivity. It is assumed that ε_j, μ_j and Σ_j are all constants, satisfying $0 < \varepsilon_j < +\infty$, $0 < \mu_j < +\infty$ and $0 \le \Sigma_j \le +\infty$. Furthermore, it is assumed that $|\varepsilon_j - 1| + |\mu_j - 1| + |\Sigma_j| > 0$ for $j = 1, 2, \ldots, l$. If $\Sigma_j = +\infty$, then Σ_j^+ is taken to be a PEC obstacle, disregarding ε_j and μ_j. In the free space, $\varepsilon = \mu = 1$ and $\Sigma = 0$. We set

$$\left(\varepsilon(x), \mu(x), \Sigma(x)\right) := \begin{cases} (\varepsilon_j, \mu_j, \Sigma_j) & \text{when } x \in \Sigma_j^+,\ j = 1, 2, \ldots, l; \\ (1, 1, 0) & \text{when } x \in \mathbb{R}^3_+ \backslash \Sigma^+. \end{cases} \tag{1.5}$$

The presence of the scatterer $(\Sigma; \varepsilon, \mu, \Sigma)$ on the ground would further perturb the propagation of the EM field $E^i - E^i_{\mathscr{G}}$, inducing the so-called *scattered* wave field E^s in $\mathbb{R}^3_+$. The scattered wave field is radiating in nature, characterized by the Silver–Müller radiation condition

$$\lim_{|x| \to +\infty} |x| \left| \left(\nabla \wedge E^s\right)(x) \wedge \frac{x}{|x|} - \mathrm{i}\omega E^s(x) \right| = 0, \tag{1.6}$$

which holds uniformly for all directions $\hat{x} := x/|x| \in \mathbb{S}^2_+$. The total electric wave field $E := E^i - E^i_{\mathscr{G}} + E^s$, together with the corresponding magnetic wave field H, is governed by the following Maxwell system

$$\nabla \wedge E - \mathrm{i}\omega\mu H = 0, \qquad \nabla \wedge H + \mathrm{i}\omega\left(\varepsilon + \mathrm{i}\frac{\Sigma}{\omega}\right)E = 0 \quad \text{in } \mathbb{R}^3_+, \tag{1.7}$$

where ε, μ and Σ are given in (1.5). Similar to (1.2), we have that

$$\nu \wedge E = \nu \wedge \left(E^i - E^i_{\mathscr{G}} + E^s\right) = 0 \quad \text{on } \mathscr{G}. \tag{1.8}$$