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Simulation of sharp interface multi-material flows involving an arbitrary number of components through an extended five-equation model

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ABSTRACT

In this paper, we present an anti-diffusive method dedicated to the simulation of interface flows on Cartesian grids involving an arbitrary number m of compressible components. Our work is two-fold: first, we introduce a m-component flow model that generalizes a classic two material five-equation model. In that way, interfaces are localized using color function discontinuities and a pressure equilibrium closure law is used to complete this new model. The resulting model is demonstrated to be hyperbolic under simple assumptions and consistent. Second, we present a discretization strategy for this model relying on a Lagrange–Remap scheme. Here, the projection step involves an anti-dissipative mechanism allowing to prevent numerical diffusion of the material interfaces. The proposed solver is built ensuring consistency and stability properties but also that the sum of the color functions remains equal to one. The resulting scheme is first order accurate and conservative for the mass, momentum, energy and partial masses. Furthermore, the obtained discretization preserves Riemann invariants like pressure and velocity at the interfaces. Finally, validation computations of this numerical method are performed on several tests in one and two dimensions. The accuracy of the method is also compared to results obtained with the upwind Lagrange–Remap scheme.

1. Introduction

The present paper deals with the simulation of compressible flows that involve m distinct materials separated by sharp interfaces on a fixed Cartesian mesh. In our physical framework, there is no velocity jump across the material front and the interfaces are passively advected by the local velocity. We suppose that all diffusive processes are negligible and that each component is equipped with a specific Equation of State (EOS).

The most straightforward simulation strategy consists in considering m subdomains with free boundaries in such way that each subdomain is occupied by a single component throughout the computation. Such method is usually called front tracking [14,28,35,61,63]. It implies implementing a tracking procedure for the interfaces and coping with the boundary conditions for each subdomain thanks to appropriate jump conditions.

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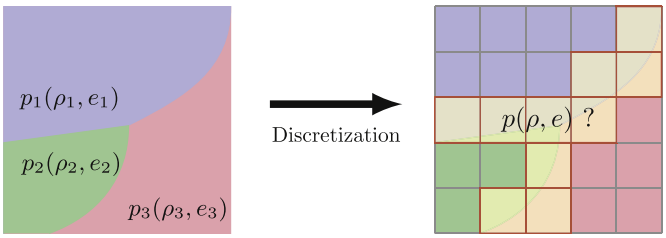

Fig. 1. Medium with multiple materials separated by interfaces. Left: the fluids are sharply separated. Right: numerical diffusion spreads the interface over few cells. A specific EOS has to be defined in this region.

We consider here another popular approach that relies on a single-fluid representation of the whole multi-material medium. The material interfaces are represented by loci of discontinuity of the medium physical properties. In our case these discontinuities will produce a switch between the various EOS of each component. Tracking these discontinuities is commonly achieved by introducing additional parameters. For example, in the well-known level set method [47] widely studied in the case $m = 2$ [22,24,43,45,46,54,64,66,67] the additional parameter is the signed distance function to the material interface. This function is then evolved using an additional Partial Differential Equation (PDE). Few works propose an extension of the level set method to treat physical situations with $m > 2$ components [66,67] and ensuring conservation properties like partial masses or momentum conservation can be a complex task.

Another approach relies on introducing discontinuous parameters $\mathcal{Z}_k$, $k = 1, \ldots, m$, often referred to as *color functions*, such that $\mathcal{Z}_k = 1$ (resp. $\mathcal{Z}_k = 0$) in regions occupied (resp. not occupied) by the sole fluid k. It is possible to use physical parameters like the mass fractions of the volume fractions as color functions. Then, the flow and the interface locations are governed by a system of PDEs formed by the evolution equation of the physical unknown parameters of the components and the m additional color functions. Unfortunately, standard discretization techniques like Finite Difference or Finite Volume methods tend to spread the discontinuities that represent the interfaces into several-grid cell wide regions due to numerical diffusion as depicted in Fig. 1. This raises two issues: first, these transition zones may not be physically relevant as our primary model was designed for modeling sharp interfaces. Second, the diffused interface regions may expand over an important portion of the computational domain. Both issues can be circumvented by implementing interfaces reconstruction techniques. This strategy has been widely used in the case $m = 2$ with the Volume of Fluid (VOF) method [33,40,53] and recent works like the Moment of Fluid (MOF) method have successfully addressed the case of $m > 2$ [26,38]. While these approaches provide a true sharp description of the interfaces in the discrete setting, they often remain complex to develop and implement.

An alternate strategy have been used in [3,4] for the case $m = 2$. In this framework, although the diffused interfaces remain *a priori* not physically relevant, they are consistent (up to a numerical truncation error) with the target sharp discontinuities. This approach belong to a family of methods that was popularized through several publications in the past years in [1–4,36,37,44,50,51,56–58,60,65]. Concerning the control of the numerical diffusion produced at the material fronts, a special Lagrange–Remap method was proposed in [39] that spares the difficulty of interfaces reconstruction. This numerical scheme encompasses an anti-diffusive discretization for updating the color functions following the lines of [17–19,41].

The present work addresses two questions: first, it proposes an extension for $m \geq 2$ of the isobaric five-equation interface capture model used in [3,4]. Second, it presents an extension of the anti-diffusive Lagrange–Remap strategy of [39] for this model. Preliminary results have been announced in [8], and we intend to provide here a detailed study of the model within a larger choice of EOS for the material components.

Before closing this introduction, we wish to refer the reader to several other significant efforts in the modeling and the simulation of multi-component flows like [10–12] by means of a phase-field description of the material interfaces and [15,41] for a study modeling possible mixture laws for of m-component flows. Moreover, we would like to mention that a similar extension of the five-equation model of [3,4] has been considered by other authors independently in [27].

The paper is structured as follows. In Section 2, we present our m-component interface capture model. Under simple hypotheses, we show that the isobaric closure is a consistent definition of a generalized EOS for the whole medium and that, granted simple thermodynamical assumptions, the overall PDE system is hyperbolic. We then detail the construction of the numerical scheme that relies on a Lagrange–Remap splitting as described in Section 3. In Section 4, we specify the anti-diffusive strategy used for the remap step, following the work of [34]. Finally we show numerical results in 1D and 2D involving up to $m = 5$ materials in Section 5.

2. The m-component interface flow model

In this part, we propose to extend the two-material five-equation model with isobaric closure [3,4] to multi-material flows with an arbitrary number of components.