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Arbitrary Lagrangian Eulerian remap treatments consistent with staggered compatible total energy conserving Lagrangian methods

J. Michael Owen [a,*], Mikhail Shashkov [b]

[a] Lawrence Livermore National Laboratory, AX Division, M/S L-38, P.O. Box 808, Livermore, CA 94550, United States
[b] Los Alamos National Laboratory, XCP-4: Methods and Algorithms, M/S F644, Los Alamos, NM 87545, Unites States

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ABSTRACT

We describe new methods of computing post-remap nodal and subzonal masses in Arbitrary Lagrangian Eulerian (ALE) calculations employing the staggered energy conserving Lagrangian hydrodynamics method of Caramana et al. (1998) [12]. An important feature of this Lagrangian algorithm is the distribution of the masses to subzonal corners within each zone, which is then used to difference the momentum and energy equations such that both linear momentum and total energy are conserved. Such algorithms present challenges when employed as part of an ALE application, however, as these subzonal masses must be treated consistently through the remap phase. In this work we develop new ideas to compute the post-remap corner masses and associated mass fluxes between the nodal control volumes, such that the new corner masses (and therefore zonal and nodal masses) are consistently defined and conservation of linear momentum is ensured through the ALE step. We demonstrate applications of these ideas on examples including pure remapping and full ALE test cases.

1. Introduction

Staggered grid Lagrangian hydrodynamics has a long history of successful application to modeling gas dynamics with multiple materials, complex flows, and strong shocks. In the late 1990s a new formulation of staggered grid Lagrangian gas dynamics was proposed [12] that could simultaneously satisfy the conservation of mass, momentum, and energy to machine precision while still performing well on problems where maintaining accurate adiabatic evolution is important. The authors refer to this approach as a compatible hydrodynamics algorithm. Compatibility as used here means that the momentum and energy equations are discretized compatibly with one and other such that total energy is conserved, or more broadly the discrete forms of the hydrodynamic equations reproduce the global invariants of their continuum counterparts: in this case the conservation of mass, linear momentum, and energy are satisfied to machine precision.

However, the Lagrangian discretization of [12] comes with its own challenges. An important property of this discretization is that it breaks up the zones of the mesh into discrete subvolumes called corners which are treated as Lagrangian – i.e., there is no mass flux in or out of these corners during a Lagrangian step. The corners represent the intersection of the primary mesh (consisting of the zonal control volumes) and the dual mesh (made up of the nodal control volumes). Fig. 1 shows one example of how these different volumes are defined for a 2D mesh consisting of hexagonal zones. The zonal and

* Corresponding author.
E-mail addresses: mikeowen@llnl.gov (J.M. Owen), shashkov@lanl.gov (M. Shashkov).

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0021-9991/

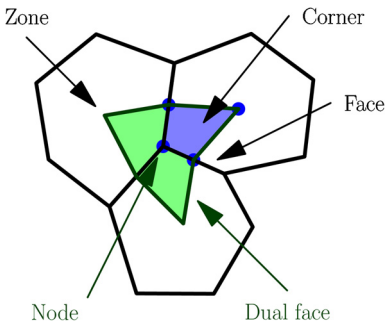

Fig. 1. Cartoon view of the primary mesh components (black) and dual mesh (green). The intersection of the primary and dual zones represent the corners described in the text. We highlight a corner in light blue here, with the corner vertices marked. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

nodal control volumes are defined as the union of the appropriate set of corners, i.e., the corners making up the zone on the primary mesh or the node on the dual. The zonal and nodal masses are the sum of their constituent corner properties, so that

$$m_z = \sum_{\text{c(zone)}} m_c, \tag{1}$$

$$m_n = \sum_{\text{c(node)}} m_c, \tag{2}$$

where m_z, m_n, and m_c represent the masses of the zone, node, and corner respectively. Note we have used the shorthand c(zone) to represent "corners of the zone" and c(node) for "corners of the node". Requiring the corners to remain Lagrangian means that both the nodal and zonal volumes are Lagrangian, which is ultimately the point of defining the corner masses in this manner.

While everything is nicely defined in terms of the Lagrangian corner volumes for the Lagrangian hydrodynamics algorithm, these definitions present us with certain difficulties in the context of ALE (Arbitrary Lagrangian Eulerian) calculations, wherein the mesh is allowed to move with a velocity other than the fluid velocity [5,6,8,19]. ALE is useful both because a Lagrangian calculation may result in an invalid mesh (such as when vorticity causes the mesh to cross over itself creating tangled control volumes), and because exerting some control over the mesh spacing can improve the accuracy in the solution. Standard ALE algorithms for staggered grid discretizations can be viewed as defining fluxes of conserved quantities between control volumes. Traditionally ALE based on a staggered grid Lagrangian method creates fluxes on the primary mesh for quantities defined on the zones (such as the thermal energy), and fluxes on the dual mesh for node centered quantities like the linear momentum. This is often implemented in the context of a HEMP Lagrangian step [29,23], which is a staggered discretization on quadrilateral elements in 2D or hexahedral elements in 3D. In HEMP the nodal masses are constructed by evenly dividing the zonal mass up between its nodes: i.e., in 2D each node gets 1/4 of the mass from each of its surrounding zones. With these definitions simple averaging of the mass fluxes from the primary mesh to the dual is adequate: the post-ALE zonal and nodal masses will be consistent, linear momentum will be preserved, etc. However, the staggered energy conserving discretization of [12] adds the complication of defining the post-ALE corner masses, which are inconsistent with the HEMP definition during the Lagrange phase. We can view the staggered discretization with corners as requiring us to define consistent mass fluxes between corners such that the total post-ALE zonal and nodal masses are reproduced. In other words, the staggered advective remap produces net fluxes of mass and momenta on the primary and dual meshes: when used with the corner based hydrodynamic algorithm we need to define consistent fluxes between the subzonal corners as well. For a purely hydrodynamic calculation the most obvious price for not reproducing nodal masses consistently following the remap stage is a loss of linear momentum conservation, which can be a serious error.

One standard approach to ALE is to split the time advancement algorithm into two distinct stages: a normal Lagrangian advance followed by a purely remapping stage, wherein the time is held fixed and the mesh moved over the material. This remapping stage will be our focus in this paper: we will assume that a Lagrangian method with corner masses is used for the Lagrangian phase of the step, and our task is to define consistent remapped properties for the zonal, nodal, and corner control volumes during the remapping phase. See [20–22] for further discussion of these issues and the algorithms discussed in this paper.

2. Definition of the remapping fluxes

If we view the transition from the initial post-Lagrange mesh to the final relaxed mesh as a linear motion of the mesh vertices, the faces of the primary mesh sweep out a volume over the original material distribution during this transition. Fig. 2 shows a cartoon of such a procedure: the black mesh represents the initial (pre-remap) configuration, the blue the