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An adjoint-based lattice Boltzmann method for noise control problems

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ABSTRACT

In this paper optimal control of acoustic problems is addressed within the lattice Boltzmann method framework. To this end, an adjoint-based lattice Boltzmann method is proposed to solve the adjoint problem. The adjoint state provides an easy access to the optimization gradients. The line search step in Newton's descent method is performed through a combination of complex differentiation and adjoint problem in the lattice Boltzmann method. The implementation of an active noise reduction method for two-dimensional weakly compressible (low Mach number) flows is discussed, and the applicability of the method is assessed.

1. Introduction

Computing the solution of a flow problem has been a motivation for research in computational fluid dynamics (CFD) since the 1940s. More recently, the computation of the derivatives of some output quantities of CFD solvers was paid a growing attention, mainly for shape optimization, flow control and sensitivity/uncertainty evaluation. In most cases, the problem can be recast as an optimization problem based on gradient methods. The derivatives of an output quantity, also known as sensitivity, can be computed by different methods (finite differences [1], adjoint problems [2], complex differentiation [3–7], automatic differentiation [8,9]). Here we focus on adjoint-based methods that require the resolution of an adjoint problem but facilitate the evaluation of the sensitivities.

The lattice Boltzmann method (LBM) (see, e.g. the book by Succi [10], Benzi et al. [11] or Aidun and Clausen [12]) is more and more used in the transports industry for aerodynamic, aeroacoustic and acoustic simulations. The two- and three-dimensional models provide accurate results for predicting aerodynamic performance and noise propagation. The method is well suited for parallel implementation, as demonstrated in [13,14] and validated implementations [15–17], and Refs. [18–20] have proven high parallel efficiency. The LBM is a good candidate as a CFD solver for weakly compressible (i.e. low Mach number) flows and therefore for flows around vehicles.

To address optimization problems, the CFD solvers are usually coupled to gradient- or non-gradient-based algorithms. Because they only require cost function evaluations, the non-gradient-based methods (like genetic algorithms) are appealing. But when dealing with a large number of parameters, they are less attractive, because they require a large number of CFD solver runs. The fitness for each individual in each population generated has to be evaluated, requiring a CFD run for each evaluation. Furthermore, genetic algorithms do not scale well with complexity. When the number of parameters increases,

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the search space size increases exponentially, reducing the benefit of genetic algorithms. Because they require only one CFD simulation and one gradient computation at each iteration, adjoint-based gradient methods are preferred when facing a large number of design parameters.

While a huge number of articles have been devoted to adjoint-based methods for Euler and Navier–Stokes equations, there are only a few papers dedicated to adjoint problems with the lattice Boltzmann method. To the knowledge of the authors, the adjoint lattice Boltzmann method (ALBM) was pioneered very recently in [21,22] while the adjoint of the lattice Boltzmann equation (LBE) was first presented in [23] with application to parameter identification in 1D and 2D flow and reused for topological shape optimization applied to 2D and 3D steady laminar flows in [1,24,25], including non-Newtonian fluids in [26]. The adjoint lattice Boltzmann method (ALBM) has recently been applied to steady-state distributed control problems in [21]. These contributions can be classified in two categories: the ALBM corresponds to a first optimize the discretize approach, as it is a method to solve the adjoint Boltzmann equation, while the adjoint of the LBE solves the adjoint equations of the already discretized problem. The terminology becomes confusing as the acronym ALBE is found in both [23] and [21] and corresponds to the adjoint lattice Boltzmann equation. In the first reference, it is obtained by building the adjoint problem to the LBE, while in the second one, it is obtained by discretizing the adjoint Boltzmann equation. As discussed further, both acronyms ALBE are similar but do not match exactly. Moreover, they correspond to a radically different approach to the optimization problem. As anteriority matters, we propose to keep ALBM for the adjoint lattice Boltzmann method and ALBE for the adjoint of the LBE. Thus, there will be an ALBM equation [21, Eq. (24)] and an ALBE method, the method used to solve the adjoint of the LBE ([23] and in the present paper).

The present paper focuses on the second approach as we propose to solve an optimization problem based on the resolution of the state by the LBM, the adjoint problem is built from the discretized (in space, time and velocity) problem obtained by the LBM. We address noise control via optimal tuning of acoustic controllers with and without base flow, i.e. unsteady aeroacoustic problem, using an adjoint problem based on the LBM. Both gradient descent and Newton's method are used. Gradient-based optimization methods are a vast research area and we present here only what is necessary for the clarity of the paper. Newton's method is easy to implement, as it only requires the evaluation of the gradient in the direction of each parameter. However, it is well known that its convergence strongly depends on the relaxation parameter and the initial guess. The gradient descent converges faster but the line search step requires the Hessian to be inverted. Both methods rely on an evaluation of the gradient of the cost function. We present here an adjoint-based method that allows straightforward access to the gradient in any direction. Computing the inverse of the Hessian is performed thanks to an original combination of the ALBE problem and complex differentiation. The adjoint of Bhatnagar–Gross–Krook lattice Boltzmann equation (BGK–LBE) is presented and the differences with the discretization of the adjoint of the Boltzmann equation are discussed. While the work by Tekitek et al. was restricted to linear collision operator, we address the fully non-linear case. Also, the application to noise control is new and we provide an original way to evaluate the Hessian in Newton-like methods.

The paper is organized as follows. In Section 2 the gradient-based optimization methods are recalled and the adjoint problem is introduced. In Section 3 the adjoint of the lattice Boltzmann equation is detailed, following a first discretize then optimize path. We believe this is the method to solve the adjoint problem arising from an LBM simulation. The differences between the ALBE and the ALBM equations are presented at the end of Section 3. Lastly, Section 4 provides some examples of acoustic optimization problems solved using the LBM and the ALBE methods. Finally the paper is concluded in Section 5 and perspectives are given.

2. Adjoint-based strategies for optimization problems

In this section we briefly recall the formulation of adjoint problems for gradient based optimization methods. In a very general way, the goal is to optimize (i.e. to minimize) a cost function I_0 that depends on the flow and some control parameters. As we will use LBM as a CFD solver, the flow variables are the populations noted f_i. The control parameters are denoted $\boldsymbol{\alpha}$, each component α_a being one independent parameter. Both the cost function and the flow variables depend on the control parameters. Therefore, the optimization problem is to find $\boldsymbol{\alpha}_{\mathrm{opt}}$ that makes I_0 minimal:

$$\boldsymbol{\alpha}_{\mathrm{opt}} = \arg\min I_0\big(f(\boldsymbol{\alpha}), \boldsymbol{\alpha}\big). \tag{1}$$

The two methods discussed below are iterative methods. From a initial guess $\boldsymbol{\alpha}_0$, a sequence of parameters $\boldsymbol{\alpha}_h$ is sought, such that $I_0(f(\boldsymbol{\alpha}_{h+1}), \boldsymbol{\alpha}_{h+1}) \le I_0(f(\boldsymbol{\alpha}_h), \boldsymbol{\alpha}_h)$.

2.1. Gradient descent

The best direction to have a decreasing I_0 is the opposite of the gradient of I_0. Thus, at each iteration one obtains a new set of parameters:

$$\boldsymbol{\alpha}_{h+1} = \boldsymbol{\alpha}_h - r_h \nabla I_0(\boldsymbol{\alpha}_h), \tag{2}$$

with r_h a relaxation parameter that is allowed to change at each iteration. It can be computed with a line search in the direction of $\nabla I_0(\boldsymbol{\alpha}_h)$. A Taylor series expansion of I_0 around $\boldsymbol{\alpha}_h$ gives a good estimation of r_h: