Journal of Computational Physics 276 (2014) 291–314

Contents lists available at ScienceDirect

Journal of Computational Physics

www.elsevier.com/locate/jcp

Incompressible SPH method based on Rankine source solution for violent water wave simulation

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ARTICLE INFO

Article history:
Received 19 July 2013
Received in revised form 20 July 2014
Accepted 22 July 2014
Available online 29 July 2014

Keywords:
Meshless method
SPH
ISPH
ISPH_R
Free surface flow
Wave impact
Violent water waves

ABSTRACT

With wide applications, the smoothed particle hydrodynamics method (abbreviated as SPH) has become an important numerical tool for solving complex flows, in particular those with a rapidly moving free surface. For such problems, the incompressible Smoothed Particle Hydrodynamics (ISPH) has been shown to yield better and more stable pressure time histories than the traditional SPH by many papers in literature. However, the existing ISPH method directly approximates the second order derivatives of the functions to be solved by using the Poisson equation. The order of accuracy of the method becomes low, especially when particles are distributed in a disorderly manner, which generally happens for modelling violent water waves. This paper introduces a new formulation using the Rankine source solution. In the new approach to the ISPH, the Poisson equation is first transformed into another form that does not include any derivative of the functions to be solved, and as a result, does not need to numerically approximate derivatives. The advantage of the new approach without need of numerical approximation of derivatives is obvious, potentially leading to a more robust numerical method. The newly formulated method is tested by simulating various water waves, and its convergent behaviours are numerically studied in this paper. Its results are compared with experimental data in some cases and reasonably good agreement is achieved. More importantly, numerical results clearly show that the newly developed method does need less number of particles and so less computational costs to achieve the similar level of accuracy, or to produce more accurate results with the same number of particles compared with the traditional SPH and existing ISPH when it is applied to modelling water waves.

1. Introduction

Smoothed Particle Hydrodynamics (SPH) is a Lagrangian meshless particle method. It was originally developed to simulate astrodynamics [1,2] but has been extended to model dynamics problems with violent motions in many areas [3–23]. The work on the method has been continuously reviewed by many authors. We mainly take some of these related to water wave modelling here.

When the SPH is applied to modelling water waves, there are largely two different formulations in literature. The first one is weakly compressible SPH (WCSPH, also called traditional SPH in this paper), in which water is considered as slightly compressible and its pressure is related to its density through an equation of state with artificially specified sound speed [6].

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http://dx.doi.org/10.1016/j.jcp.2014.07.036
0021-9991/

The second formulation is incompressible SPH, also called ISPH, in which water is considered as incompressible and having constant density with pressure found by solving a boundary value problem. As indicated by many researchers, e.g., Rafiee et al. [13] and Lee et al. [22], the WCSPH has several advantages, such as that it is easy to be programmed and does not need to solve pressure boundary value problem. However, it has at least two weaknesses [13,22]: (a) requiring use of very small time steps and (b) resulting in significant spurious pressure fluctuations in space and time domain. The first one is inherent because the sound speed in the equation of state has to be large enough (even though much smaller than real sound speed), and so leads to smaller time steps. This is also related to that fact that the traditional SPH usually needs a large number of particles; in other words, very small distance between particles or very high resolution to obtain sufficiently accurate results. The more the particles in a given computational domain, the smaller the time steps should be used. The second weakness is perhaps because the pressure is very sensitive to the density and so a small error in density can lead to a significant error in pressure, inducing spurious pressure fluctuations. The spurious pressure fluctuations do not only give wrong values of pressure but can also cause simulation to become unstable when the method is employed to model wave–structure interactions. A great amount of effort has been made to overcome the weaknesses, for example, use of K2_SPH [8] and SPH based on the solution for Riemann problem [9–12]. K2_SPH improves the accuracy of kernel approximation for partial derivatives in continuity equation and momentum equations, and so can give more accurate results of density and velocity. In the SPH based on the solution for a Riemann problem, the Riemann problem is solved for each pair of particles to calculate the associated parameters. Basically, this approach improves accuracy of estimating the gradient involved. Its results are much more accurate and smoother than these from the traditional SPH, but it is significantly more expensive (in the order of 5 or 6 times) than the latter [13].

The ISPH has also been widely applied in the field of water wave dynamics [15–23]. This method projects the intermediate velocity field onto a divergence-free space by solving a Poisson equation for pressure. According to comparative studies carried out by Lee et al. [22], the time step used for the ISPH can be much larger (50 times larger in one of cases presented by them). In addition, the results from ISPH can be much more accurate than these from the WCSPH for a given number of particles. In other words, the convergence rate of ISPH results is much higher than that of the WCSPH. The drawback of this formulation is obvious as it needs to solve the boundary value problem defined by the Poisson equation at each time step, which is recognised to consume a significant amount of computational time. Nevertheless, the total computational time taken by the ISPH can be shorter than that by the WCSPH, as indicated also by Lee et al. [22]. However, the second order derivatives of pressure need to be approximated when discretising the Poisson equation. In all publications found so far in literature about ISPH, the second derivatives are directly approximated using a scheme similar to that for finite difference method. No matter what scheme to be used, direct numerical approximation to second derivatives always has a difficulty with accurately modelling the functions to be solved, in particular when particles are distributed in a disorderly manner. Distribution of particles always becomes disorderly when modelling violent waves even they are regularly distributed at the start of simulation. Therefore, it is obviously advantageous to eliminate use of direct numerical approximation to second derivatives when solving the pressure Poisson equation in the ISPH formulation.

The distinct feature of this paper, compared with other papers on ISPH lies in that the pressure Poisson equation is first transformed into another equation based on a Rankine source solution using the same idea employed in Meshless Local Petrov–Galerkin Method based on Rankine Source Solution (MLPG_R) [24–30]. In the new formulation of ISPH, the governing equation for pressure does not include any derivatives of the functions to be solved and so overcomes the problems associated with direct numerical approximation to second derivatives in existing ISPH formulation. This new formulated ISPH is named as ISPH_R for convenience in this paper. According to our benchmark tests presented in this paper below, the ISPH_R method can give more accurate results and consume less computational time when modelling water waves.

2. Governing equations and numerical schemes

2.1. Traditional SPH method

The formulation of the traditional SPH can be found in many publications but it will be outlined in this section for completeness. The method is generally based on the Lagrangian form of continuity equation and the Navier–Stokes equation for compressible flow, which may be written as

$$\frac{D\rho}{Dt} + \rho\nabla\cdot\mathbf{u} = 0 \tag{1}$$

$$\frac{D\mathbf{u}}{Dt} = -\frac{1}{\rho}\nabla p + \mathbf{g} + \nu\nabla^2\mathbf{u} \tag{2}$$

where ρ is the fluid density, $\mathbf{u}$ is the fluid velocity, t is the time, p is the fluid pressure, $\mathbf{g}$ is the gravitational acceleration, and ν is the kinematic viscosity. In WCSPH, the pressure and density are usually related by the following equation of state for sound waves

$$p = \frac{c_0^2\rho_0}{\kappa}\left[\left(\frac{\rho}{\rho_0}\right)^{\kappa} - 1\right] \quad \text{or} \quad p = c_0^2(\rho - \rho_0) \tag{3}$$