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Hybrid simulation of whistler excitation by electron beams in two-dimensional non-periodic domains

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ABSTRACT

We present a two-dimensional hybrid fluid-PIC scheme for the simulation of whistler wave excitation by relativistic electron beams. This scheme includes a number of features which are novel to simulations of this type, including non-periodic boundary conditions and fresh particle injection. Results from our model suggest that non-periodicity of the simulation domain results in the development of fundamentally different wave characteristics than are observed in periodic domains.

1. Introduction

Whistlers are electromagnetic waves that exist in a magnetized plasma below the electron cyclotron frequency, $\Omega = eB_0/m_e$ (where e is the fundamental charge, m_e is the electron mass, and B_0 is the magnitude of the background magnetic field, $\mathbf{B}_0$). There is great interest in both naturally occurring and artificially induced whistlers because of their ability to resonantly interact with energetic electrons. It is believed that both rapid energization and rapid loss of electrons from the terrestrial radiation belts may be due to whistler wave dynamics. In situ observations of large-amplitude whistler mode waves in the Earth's radiation belts [1–3] have sparked a renewed interest in understanding the range of possible interactions between whistlers and relativistic electrons.

Self-consistent studies of whistler-particle interactions usually take one of two forms – particle-based simulations using the particle-in-cell (PIC) method, e.g., [4], or hybrid simulations which combine a fluid description of cold background plasmas and a kinetic description (either PIC or Vlasov methods) of the energetic electron components, e.g., [5–7]. Although each approach has its strengths, hybrid methods generally allow for simulations to consider larger domains because fewer particles are needed and they are able to take larger time steps by eliminating unwanted high-frequency phenomena (e.g., plasma oscillations).

Lampe et al. [5] introduced a quasineutral hybrid simulation technique for whistler waves based on electron magnetohydrodynamics (EMHD). Although the method they described was fully valid in two dimensions, they only presented results involving particles in one dimension. In this paper, we extend those results to two dimensions and discuss some of the complications that are encountered when using such a model in two dimensions, particularly when avoiding the use of periodic boundary conditions.

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There is a robust theoretical literature regarding whistler-electron interactions, and a comprehensive discussion of it can be found in Shklyar & Matsumoto [8]. An interesting aspect of this interaction is that whistlers and electrons have a counter-propagating resonance, requiring wave propagation in the opposite direction of the resonant particle, since

$$\omega - (\mathbf{k}\cdot\mathbf{v})_{\parallel} = \frac{n\Omega}{\gamma} \tag{1}$$

where ω is the wave frequency, n is the harmonic number, $\mathbf{k}$ is the wave vector, $\mathbf{v}$ is the particle velocity, γ is the relativistic Lorentz factor, and the subscript $\parallel$ indicates the component along the background magnetic field.

In this paper, we will describe a two-dimensional hybrid fluid-PIC model for whistler wave interaction with relativistic electrons. We will begin by providing theoretical background for the EMHD model and the beam-whistler instability. Next, we will discuss some of the numerical and algorithmic challenges that have been dealt with in the construction of this model. Finally, we will demonstrate the excitation of whistlers by beams finite beam dimensions, noting the presence of a characteristic sequence of wave growth "phases" that precede saturation.

2. The hybrid electron magnetohydrodynamics model

Electron magnetohydrodynamics is an approximation suitable for modeling plasma wave phenomena in the frequency range $\omega_{LH} < \omega < \Omega$, where ω_{LH} is the lower hybrid frequency, Ω is the electron cyclotron frequency, and ω is the wave (angular) frequency. In this approximation, the plasma is assumed to be quasineutral ($n_i \approx n_e \equiv n$) and ions are taken to be an immobile neutralizing background.

In our hybrid version of the EMHD model, the primary equations are Faraday's law, Ampere's law (without displacement current), and Ohm's law

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla\times\mathbf{E} \tag{2}$$

$$\mathbf{u} = -\frac{1}{n_c e}\left(\frac{\nabla\times\mathbf{B}}{\mu_0} - \mathbf{J}_h - \mathbf{J}_s\right) \tag{3}$$

$$\mathbf{E} + \lambda_e^2\nabla\times\nabla\times\mathbf{E} = -\mathbf{u}\times\mathbf{B} - \frac{m_e}{n_c e}\left(n_c\nu\mathbf{u} + \nabla\cdot(n_c\mathbf{u}\mathbf{u}) + \frac{1}{e}\left(\frac{\partial \mathbf{J}_h}{\partial t} + \frac{\partial \mathbf{J}_s}{\partial t}\right)\right) \tag{4}$$

where $\lambda_e = c/\omega_e$ is the electron inertial length, ν is the collision frequency, and the subscripts c, h, and s refer to cold electrons, hot electrons, and external sources, respectively.

In both Ampere's law and Ohm's law, there are terms involving the current carried by hot electrons, $\mathbf{J}_h$. We model the evolution of the hot electron population using a particle-in-cell (PIC) approach based on the methods of Birdsall & Langdon [9]. The trajectory of the ith individual particle is determined by the Newton–Lorentz force law,

$$\frac{\partial \mathbf{p}_i}{\partial t} = -e(\mathbf{E} + \mathbf{v}_i\times\mathbf{B}) \tag{5}$$

$$\frac{\partial \mathbf{x}_i}{\partial t} = \mathbf{v}_i \tag{6}$$

Note that we are allowing for relativistic particles, so $\mathbf{p}_i = m_e\gamma_i\mathbf{v}_i$, where $\gamma_i^{-1} = \sqrt{1 - v_i^2/c^2}$ is the particle's Lorentz factor.

Each particle is assigned a weight, w_i, such that the sum over all particle weights within a grid cell is equal to the average particle density, $n_h = \sum_{i=1}^{m} w_i$, in that grid cell, where m is the number of particles in that cell. Similarly, the hot particle current density contribution is determined by summing over the product of the weights and velocities of particles in the cell, $\mathbf{J}_h = -e\sum_{i=1}^{m} w_i\mathbf{v}_i$. Note that w_i is taken to be a constant in our runs, but it may in principle be allowed to vary.

3. Numerical solution of the EMHD equations

We simulate the evolution of the hybrid EMHD system in a rectangular $L_x \times L_z$ domain. We discretize this domain into $n_x \times n_z$ grid cells. As shown in Fig. 1, the electric field and cold plasma velocity fields are collocated on the grid corners while the magnetic fields are collocated on the cell centers. We approximate all spatial derivatives using second order central differences.

3.1. The cold plasma current

Because of the spatial arrangement between the cold plasma velocity and magnetic fields, it is necessary to interpolate the magnetic fields before obtaining the fluid velocity from Eq. (3). It should be noted that the use of Ampere's law implies that $\nabla\cdot\mathbf{J} = \nabla\cdot(\mathbf{J}_c + \mathbf{J}_h) = 0$, but because of the configuration of grid cells, the finite difference approximation does not guarantee that this condition will actually be satisfied. Thus we explicitly correct for the possibly divergent part of the current in order to maintain quasineutrality: