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A staggered-grid finite-difference scheme optimized in the time–space domain for modeling scalar-wave propagation in geophysical problems

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ABSTRACT

For modeling scalar-wave propagation in geophysical problems using finite-difference schemes, optimizing the coefficients of the finite-difference operators can reduce numerical dispersion. Most optimized finite-difference schemes for modeling seismic-wave propagation suppress only spatial but not temporal dispersion errors. We develop a novel optimized finite-difference scheme for numerical scalar-wave modeling to control dispersion errors not only in space but also in time. Our optimized scheme is based on a new stencil that contains a few more grid points than the standard stencil. We design an objective function for minimizing relative errors of phase velocities of waves propagating in all directions within a given range of wavenumbers. Dispersion analysis and numerical examples demonstrate that our optimized finite-difference scheme is computationally up to 2.5 times faster than the optimized schemes using the standard stencil to achieve the similar modeling accuracy for a given 2D or 3D problem. Compared with the high-order finite-difference scheme using the same new stencil, our optimized scheme reduces 50 percent of the computational cost to achieve the similar modeling accuracy. This new optimized finite-difference scheme is particularly useful for large-scale 3D scalar-wave modeling and inversion.

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1. Introduction

Finite-difference (FD) schemes have been widely used for simulations of wave propagation (e.g., [1]). High-order FD schemes are particularly attractive for large-scale 3D modeling, because they are able to control numerical dispersion using a larger grid spacing compared with low-order schemes (e.g., [2,3]). The coefficients of high-order FD operators are usually determined using the Taylor expansion of the truncation error ε with respect to the grid spacing h such that $\varepsilon = O(h^{2M})$, where $2M$ is the length of the standard FD operator or stencil (e.g., [4]). Equivalently, one may express the phase-velocity error ϵ_v in terms of the normalized wavenumber "kh" and design the coefficients such that $\epsilon_v = O((kh)^{2M})$ (e.g., [5]), where k is the wavenumber. Although very high-order accuracy has been achieved in space, second-order time discretization is popular for modeling large-scale wave propagation because of its relatively low requirements of computer memory (e.g., [6]).

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In the following, we refer the scheme with $2M$th-order accuracy in space and second-order accuracy in time as the standard high-order FD scheme.

The numerical solution obtained using a high-order scheme converges rapidly to the true solution when the grid spacing h approaches zero. In other words, high-order schemes are excellent at controlling the phase-velocity error for low wavenumbers. However, for a given h, high-order schemes do not always sufficiently suppress the error for large wavenumbers. One remedy is increasing the length of the FD operator $2M$, leading to high computational costs. When $2M$ approaches the total number of grid points along one direction, high-order FD schemes essentially become the pseudospectral method [7,8], which is free of numerical dispersion but is computationally expensive.

Holberg [9] introduced an optimized FD scheme to control spatial dispersion errors for a wide range of wavenumbers for a given length of the FD operator for numerical modeling of seismic-wave propagation. His objective function for optimization is the maximum relative error of the group velocity. Various optimized FD schemes have emerged since Holberg's pioneering work.

Numerous optimized FD schemes minimize the spatial dispersion error and temporal dispersion error independently (e.g., [10–14]). Lele [10] obtained compact FD schemes with spectral-like resolution by imposing the requirement that the discrete FD operator matches the spatial differential operator at three prescribed high wavenumbers. Tam and Webb [11] constructed a dispersion–relation-preserving (DRP) scheme by optimizing the FD approximations to the spatial and temporal differential operators. Bogey and Bailly [12] advanced the DRP scheme using spatial FD operators with a length of up to 13 grid points. Hu et al. [13] developed low-dissipation and low-dispersion Runge–Kutta time-advancing schemes. Zhang and Yao [14] found that the norm of objective functions plays an important role for designing optimized FD schemes. Their schemes based on the maximum norm have more flexibility and better accuracy than those in [11,12] based on L^2- or L^1-norm.

Several optimized FD schemes minimize the spatial dispersion error and temporal dispersion error simultaneously. Haras and Ta'asan [15] minimized the global truncation error of the partial differential equation, and demonstrated that their compact FD scheme is more accurate than Lele's spectral-like scheme in [10] for solving the scalar-wave equation. Etgen [16] developed an FD scheme to minimize the phase-velocity error. His scheme balances both spatial and temporal dispersion errors because the two types of errors have opposite signs. Stork [17] designed spatial FD operators that vary between consecutive time steps to reduce dispersion errors. Liu [18] found that minimizing the relative error of the time–space-domain dispersion relation can lead to smaller relative errors of the phase velocity, compared to minimizing the absolute error of the space-domain dispersion relation. He obtained his globally optimal FD schemes by linearizing an objective function of the relative phase-velocity error and solving it using a least-squares approach.

The FD schemes optimized in time–space domain in [9,16–18] are based on the standard finite-difference stencil composed of grid points on the axis along which the spatial derivative is calculated. Liu and Sen [5] demonstrated that high-order FD schemes based on the standard $2M$-point stencil can reach the $2M$th-order accuracy both in space and time, but only along eight directions of wave propagation in 2D and 48 directions in 3D when using wavefields at one time step for temporal evolution. The temporal accuracy is still second order along the other propagation directions. As high-order FD schemes, optimized schemes based on the standard stencil still have low temporal accuracy when using wavefields at one time step for temporal evolution, and a small time interval has to be used to adequately control temporal dispersion errors.

We recently developed a new staggered-grid finite-difference method in the time–space domain to improve the accuracy in time [19]. The stencil, same as that for Lax–Wendroff scheme (e.g., [2,20,21]), contains a few additional grid points off each axis compared to the standard stencil. Our new FD scheme increases the temporal accuracy from second order to fourth order for FD modeling with high-order spatial accuracy. The computer-memory requirement of our FD scheme is similar to that of the standard high-order FD scheme with second-order accuracy in time.

In this paper, we develop an optimized staggered-grid finite-difference scheme in the time–space domain based on our new stencil for solving 2D and 3D scalar-wave equations. Scalar-wave equations are widely used in important geophysical problems, including reverse-time migration [22] and full-waveform inversion [23]. In such problems, the phase error is one of the major concerns. Our objective function for optimization is thus the relative error of the phase velocity for waves propagating in all directions within a given range of wavenumbers. Our optimized scheme not only suppresses spacial dispersion errors for large wavenumbers, but also allows us to use a large time interval and well control time dispersion errors. The advantage of using a large time interval for numerical wave modeling highlights the novelty of our FD scheme. We preform dispersion analysis for our optimized FD scheme, and use our optimized scheme to conduct numerical modeling of scalar-wave propagation in 2D and 3D complex media. Our results demonstrate that the computational efficiency of our optimized FD scheme is up to 2.5 times higher than that of the optimized schemes based on the standard stencil for a given 2D or 3D modeling problem. Compared with the high-order FD scheme based on the same new stencil, our optimized scheme achieves the same modeling accuracy with only a half of the original computational cost.

We design our optimized FD scheme specifically for modeling scalar-wave propagation in geophysical problems, as in [9,14,16–18]. The development of the optimized FD schemes in [10–13,15] is motivated by computational fluid dynamics (CFD) problems. Although the two types of problems are both governed by hyperbolic equations, they have their own computational challenges for practical applications. For example, the spectrum content of the waves and propagation distances may be different. Moreover, different boundary conditions are imposed for geophysical problems and CFD problems. For geophysical problems, two types of boundary conditions are usually imposed for the scalar-wave equation: absorbing boundary conditions and free-surface boundary conditions (e.g., [1,24]). The former is used to truncate an unbounded domain (the Earth) into a bounded one. There have been extensive studies of absorbing boundary conditions for modeling seismic-wave